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Optimal harvest age considering multiple carbon pools – a comment

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Abstract:
In two recent papers, Asante and Armstrong (2012) and Asante et al. (2011) considered the question of optimal harvest ages. They found that the larger are the initial pools of dead organic matter (DOM) and wood products, the shorter is the optimal rotation period. In this note, it is found that this conclusion follows from the fact that the authors ignored all release of carbon from decomposition of DOM and wood products after the time of the first harvest. When this is corrected for, the sizes of the initial stocks of DOM and wood products do not influence the optimal rotation period. Moreover, in contrast to the conclusions in the two mentioned papers, our numerical analysis indicates that inclusion of DOM in the model leads to longer, not shorter, rotation periods.

Keywords: Dead organic matter, forestry, Faustmann, carbon

JEL classification: Q23, Q54, Q42

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Introduction

Concerns related to the accumulation of CO_2 in the atmosphere have given rise to studies on how the carbon pools of forests should influence forest management; see, for example, Haberl et al. (2012a,b), Hoel et al. (2012), Holtsmark (2012a,b), McKechnie et al. (2011), Price and Willis (2011), Schulze et al. (2012), Searchinger et al. (2009), Tahvonen (1995), and van Kooten et al. (1995).

In two recent contributions to this research field, Asante et al. (2011) and Asante and Armstrong (2012) studied the question of optimal rotation age when carbon pools such as dead organic matter (DOM) and wood products were included in their model. They reached the conclusion that the larger is the initial size of these carbon pools, the shorter is the optimal rotation period. More generally, they found that if DOM as a carbon stock was included in their analysis, there was a tendency towards a shorter rotation age.

Their first mentioned finding with regard to the initial size of the carbon pools of DOM and wood products is surprising. The time profile of the release of CO_2 emissions from these pools is determined by these pools' initial sizes and their speeds of decomposition, but is not influenced by the harvest age. It is therefore difficult to understand these results. However, for the purpose of mathematical simplicity, they considered a single rotation period only. Moreover, with a time perspective strictly limited to the first rotation cycle, they did not take into account the release of CO_2 from the initial pools of either wood products or DOM *after* the time of the first harvest. Consequently, the shorter is the rotation cycle, the smaller are the accounted emissions from these pools, with their method. In this paper, we will show that, when we account for the release of carbon from the initial pools of DOM and wood products during and after the time of the first harvest, the initial sizes of these carbon pools do not matter with regard to optimal harvest age. Moreover, for similar reasons, their conclusions with regard to the effect of including DOM and wood product pools in general in the model are not confirmed. Our numerical analysis indicates that inclusion of DOM in the model leads to *longer*, not shorter, rotation periods.

Theoretical framework and result

The model

To make our analysis comparable, we adopt the theoretical approach of Asante and Armstrong (2012), considering a single rotation period only. However, as we nevertheless will take into consideration decomposition of DOM and wood products after the time of the first harvest, some adjustments of their model are required. We will return to this issue later in our paper.

Let W_V be the net present value of the net income from harvest from the considered forest stand. Following Asante and Armstrong (2012), we have:

$$W_V(T,p) = (pV(T) - C^a)\, e^{-\rho T}, \qquad (1)$$

where p is the net price of timber (gross price of timber minus harvest costs per unit volume) measured in monetary units per tC, $V(T)$ is the timber volume at the time of harvest T, measured in tonnes of carbon per ha (tC/ha), C^a is a fixed harvest cost, and ρ is the discount rate. We assume that $V'(t) > 0$ and that $V(0) = 0$. To simplify our notation, we measure all variables with regard to their carbon content.

Next, assume that there is a social cost related to carbon emissions, p_C (measured in monetary units per tC). Let $B(t)$ be the stock of living biomass on the stand, assuming that $B(t) > V(t)$, $B'(t) > 0$ and that $B(0) = 0$. The net present value of carbon sequestration in living biomass over the considered rotation period is then:

$$W_B(T,p_C) = p_C \int_0^T e^{-\rho t} B'(t)\, dt - p_C e^{-\rho T} B(T). \qquad (2)$$

The first term on the right-hand side represents the value of carbon sequestration in the forest over the first rotation cycle, whereas the second term represents the costs of the removal of the pool of living biomass at the time of harvest. Note, however, that a share of the living biomass, $B(T) - V(T)$, at time T, is transferred to the pool of residues. This is taken care of by the first term in equation (5).

In contrast to Asante and Armstrong (2012), we distinguish between *two* pools of DOM: naturally generated dead organic matter (NDOM), $D(t)$, and harvest residues, $R(t)$. The development of the latter carbon pool is not explicitly considered by Asante and Armstrong (2012) because they did not study the development of *any* carbon pools after the time of the harvest.

Consider then emissions from decomposition of harvest residues. At the time of harvest, a stock $V(T)$ of biomass is removed from the forest. The remaining living biomass is transferred to the pool of harvest residues. Hence, a stock of harvest residues $R(T) = B(T) - V(T)$ is generated. We assume that harvest residues decompose at the rate α, as does NDOM. Hence, at time $t \geq T$, we have:

$$R(t) = e^{-(t-T)\alpha} R(T), \qquad (3)$$

$$R'(t) = -\alpha e^{-(t-T)\alpha} R(T). \qquad (4)$$

If $t < T$, then $R(T) = 0$. The net present social value of harvest residues from the first harvest is:

$$W_R(T,p_C) = p_C e^{-\rho T} R(T) - p_C \int_T^\infty \alpha e^{-\rho t} e^{-\alpha(t-T)} R(T)\, dt. \qquad (5)$$

The first term on the right-hand side of (5) represents the social value of the generation of residues from the first harvest. This must be related to the second term on the right-hand side of (2), as explained below. The second term of (5), which is left out of the analysis by Asante and Armstrong (2012) and Asante et al. (2011), follows directly from (4) and represents the discounted costs of all future release of carbon as the residues decompose. Equation (5) could be simplified to:

$$W_R(T, p_C) = p_C e^{-\rho T} \frac{\rho}{\rho + \alpha} R(T). \tag{6}$$

With regard to DOM, Asante and Armstrong (2012) and Asante et al. (2011) emphasized the importance of the initial stock, as mentioned. We therefore distinguish between DOM generated before $t = 0$, and DOM generated later. We label the remaining part of the initial stock of DOM $D_I(t)$, which could include residues from previous harvest events, whereas the stock of NDOM generated after $t = 0$ is labelled $D(t)$. It follows that $D_I(0) \geq 0$, while $D(0) = 0$. The remaining share of the initial stock of DOM develops as follows:

$$D_I'(t) = -\alpha D_I(t). \tag{7}$$

Hence, at time t, the remaining share of this stock is:

$$D_I(t) = e^{-t\alpha} D_I(0). \tag{8}$$

The time derivative is:

$$D_I'(t) = -\alpha e^{-t\alpha} D_I(0), \tag{9}$$

and the net present social costs of all future release of carbon from the initial DOM pool are then:

$$C_{D_I}(p_C, D_I(0)) = p_C \int_0^\infty \alpha D_I(0) e^{-t(\rho+\alpha)} dt. \tag{10}$$

This expression is simplified to:

$$C_{D_I}(p_C, D_I(0)) = p_C \frac{\alpha}{\rho + \alpha} D_I(0). \tag{11}$$

Next, consider NDOM generated after $t = 0$. This pool develops as follows:

$$D'(t) = \beta B(t) - \alpha D(t), \tag{12}$$

where β is a positive parameter and the term $\beta B(t)$ represents litterfall, whereas α is defined above such that $\alpha D(t)$ represents decomposition. This means that the amount of NDOM generated at time k that is left at time t is $e^{-(t-k)\alpha} \beta B(k)$. The stock of NDOM (exclusive of the remaining share of the initial stock of DOM) is then:

$$D(t) = \begin{cases} e^{-t\alpha} \int_0^t e^{k\alpha} \beta B(k)\, dk, & t \in (0, T], \\ e^{-(t-T)\alpha} D(T), & t \in (T, \infty). \end{cases} \tag{13}$$

The time derivative of NDOM is then:

$$D'(t) = \begin{cases} \beta B(t) - \alpha e^{-t\alpha} \int\limits_0^t e^{-k\alpha} \beta B(k)\, dk, \ t \in (0, T], \\ -\alpha e^{-(t-T)\alpha} D(T), \ t \in (T, \infty), \end{cases} \tag{14}$$

where the first term on the first line represents the generation of naturally dead organic matter whereas the second term in the first line represents decomposition of the NDOM pool. The net present value of the NDOM that is generated during the first rotation period is:

$$\begin{aligned} W_D(T, p_C) = {} & p_C \int\limits_0^T e^{-t\rho} \left(\beta B(t) - \alpha e^{-t\alpha} \int\limits_0^t e^{-k\alpha} \beta B(k)\, dk \right) dt \\ & - p_C \int\limits_T^\infty e^{-t\rho} \alpha e^{-(t-T)\alpha} D(T)\, dt. \end{aligned}$$

The first line in this equation represents the social discounted value related to the net accumulation of NDOM before harvest, and follows directly from the first line of (14). The second line represents the discounted social costs of the release of carbon from decomposition of the NDOM pool after the harvest, and follows from the second line of (14). In Asante and Armstrong (2012) and Asante et al. (2011), the latter term was left out of the analysis.

Finally, we consider the wood product pool. Asante and Armstrong (2012) focused on the size of the initial product pool and found that the size of this pool influences the optimal length of the rotation age. Corresponding to the treatment of the initial stock of DOM, we therefore define an initial stock of wood products labelled $Z_I(0)$. The discounted cost of the release of carbon from decomposition of the initial product pool is:

$$C_{Z_I}(p_C, Z_I(0)) = p_C \int\limits_0^\infty \theta Z_I(0)\, e^{-t(\rho+\theta)} dt, \tag{15}$$

where θ is the decay rate of wood products. The expression in (15) is simplified to:

$$C_{Z_I}(p_C, Z_I(0)) = p_C \frac{\theta}{\rho + \theta} Z_I(0). \tag{16}$$

Consider next the stock of products with its origin in the first harvest, which we label $Z(t)$. As in Asante and Armstrong (2012), we assume that a share λ of the harvest is transferred to the product pool. When $t > T$, this pool develops as follows:

$$Z(t) = e^{-(t-T)\theta} \lambda V(T). \tag{17}$$

The time derivative is:

$$Z'(t) = -\theta e^{-(t-T)\theta} \lambda V(T). \tag{18}$$

The net present value of the carbon contained in the product pool from the first harvest is then:

$$W_Z(T, p_C) = p_C \left(e^{-T\rho} \lambda V(T) - \theta \lambda e^{T\theta} V(T) \int_T^{\infty} e^{-t(\rho+\theta)} dt \right), \tag{19}$$

where the first term within the outer parentheses represents the enlargement of the product pool at the time of harvest, whereas the second term represents discounted values of all future releases of carbon due to use of the wood products as an energy source.

Summing up, all elements of the net social welfare generated by the first harvest cycle, $W(p, T, p_C)$, are then:

$$\begin{aligned} W(p, T, p_C) &:= W_V(\cdot) + W_B(\cdot) + W_R(\cdot) \\ &\quad + W_D(\cdot) - C_{D_I}(\cdot) + W_Z(\cdot) - C_{Z_I}(\cdot). \end{aligned} \tag{20}$$

As mentioned, we have adopted the single rotation approach to make our results comparable to those of Asante and Armstrong (2012) and Asante et al. (2011). An interpretation of this approach could be that the forested area for an unspecified reason is not available for replanting after the first harvest. However, a more complete approach would consider multiple future rotations in the Faustmann tradition; see Faustmann (1849). We would prefer that approach, which we apply in Hoel et al. (2012), where we develop an adjusted Faustmann rule and present a number of theoretical and numerical results. Hence, readers interested in a more comprehensive analysis of the issue are directed to that paper.

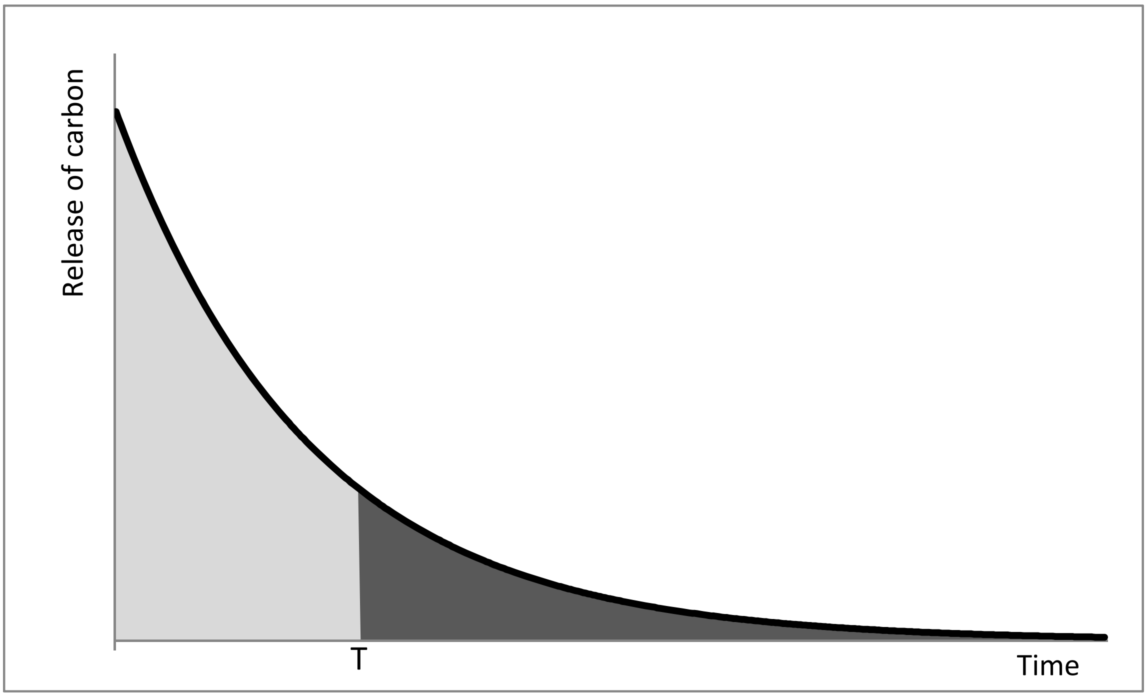
A theoretical result

In this paper, we consider a single rotation only, and present a single theoretical result.

Proposition 1. *The sizes of the initial pools of DOM and wood products do not influence the optimal time of harvest.*

Proof. From (11) and (16), it is easily seen that the social costs due to release of carbon from the initial pools of DOM and wood products are not influenced by the time of harvest. Thus, the harvest age should not be influenced by the initial pools of carbon. □

Figure 1. The line depicts the release of carbon from decomposition of an initial stock of DOM. The light grey area represents the amount of carbon released until the first harvest, whereas the dark grey area represents the amount of carbon released after the harvest



The intuition as to why Asante and Armstrong (2012) as well as Asante et al. (2011) reached the opposite conclusion is illustrated in Fig. 1. The curve in this diagram shows a possible path for decomposition of an initial stock of DOM. The size of the light grey area then gives the amount of carbon released from this pool in the period $t \in (0, T)$, which was the time horizon used in Asante and Armstrong (2012) as well as Asante et al. (2011). If T is reduced, the light grey area is reduced and, thus, so are the calculated net present social costs of all future releases of carbon from the initial DOM pool. In other words, if the release of carbon from the initial DOM pool is terminated at the time of harvest, the optimal harvest age will be decreasing in the size of the initial DOM pool. However, if the release of carbon from the initial DOM pool in the period $t \in (T, \infty)$ had also been taken into account, the net present social costs of all future releases of carbon from the initial DOM would have been fixed and independent of T. Thus, the finding of these authors is an artefact of their time limit.

Using the same argument as above with regard to the initial pool of wood products, it is obvious and easily shown that with an infinite time perspective, the size of the initial product pool is also irrelevant.

The next section briefly presents numerical simulations to illustrate how the results of Asante and Armstrong (2012) are changed when an infinite time perspective is applied.

Numerical simulations

Materials and methods

In the following, we present numerical simulations based on the model described in the previous section. Hence, we still only consider a single rotation. For corresponding simulations with a similar model with multiple rotations, see Hoel et al. (2012).

The numerical model has one-year time steps. All parameter values (see Table 1) are taken from Asante and Armstrong (2012) and are therefore not discussed here.

Asante and Armstrong (2012) considered different initial stocks of wood products and DOM. As shown in the previous section, these initial stocks are irrelevant for determination of the optimal harvest age. We therefore assume that these pools are zero in all cases considered. However, Asante and Armstrong (2012) focused more generally on the effect of including DOM in the analysis. We fully agree that this is important. Therefore, we will do the same.

The applied functional forms are as follows:

$$V(t) = v_1 \left(1 - e^{-v_2 t}\right)^{v_3}, \tag{21}$$

$$B(t) = b_1 \left(1 - e^{-b_2 t}\right)^{b_3}. \tag{22}$$

Table 1. Parameter values

v_1	100.08*	b_1	198.6	α	0.00841
v_2	0.027	b_2	0.0253	β	0.01357
v_3	4.003	b_3	2.64	ρ	0.05
C_a	6250	γ	0.2		

*This parameter is 500.4 in Asante and Armstrong (2012). As we measure timber in tC/ha instead of m^3/ha, the parameter is adjusted down to 100.08.
Source: Asante and Armstrong (2012).

Table 2. Optimal harvest age when different carbon pools are included in addition to commercial income from timber

Permit price (CAD/tCO_2)	Trunks	Trunks, other living biomass, and residues	Trunks, other living biomass, residues, and NDOM	Trunks, other living biomass, residues, NDOM, and wood products
0	69	69	69	69
1	69	70	70	70
2	70	71	72	71
5	72	74	76	76
10	77	80	86	84
20	88	97	122	110
30	108	130	∞	∞

Figure 2. Optimal harvest ages when the social values of different carbon pools are included in the analysis

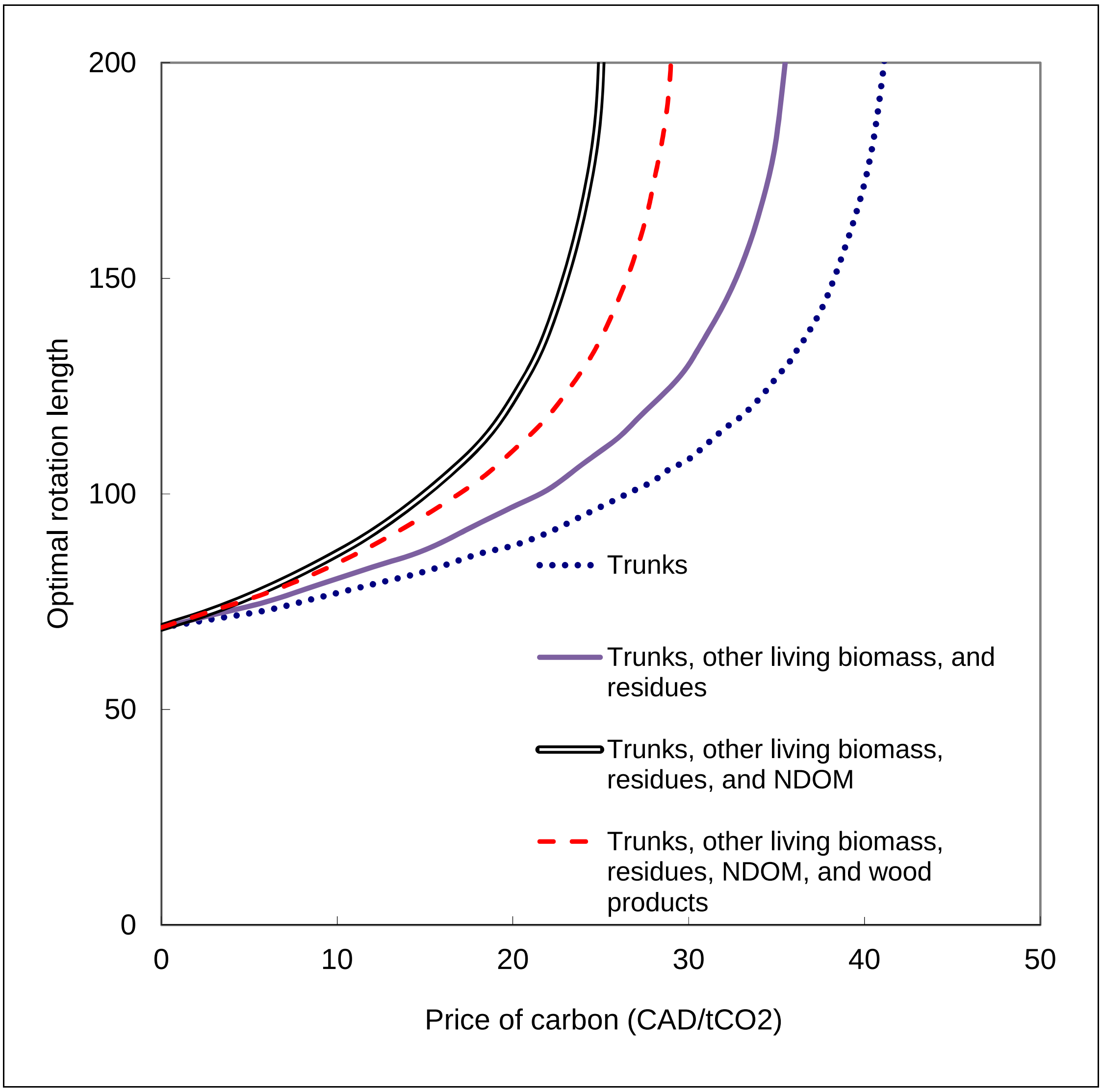
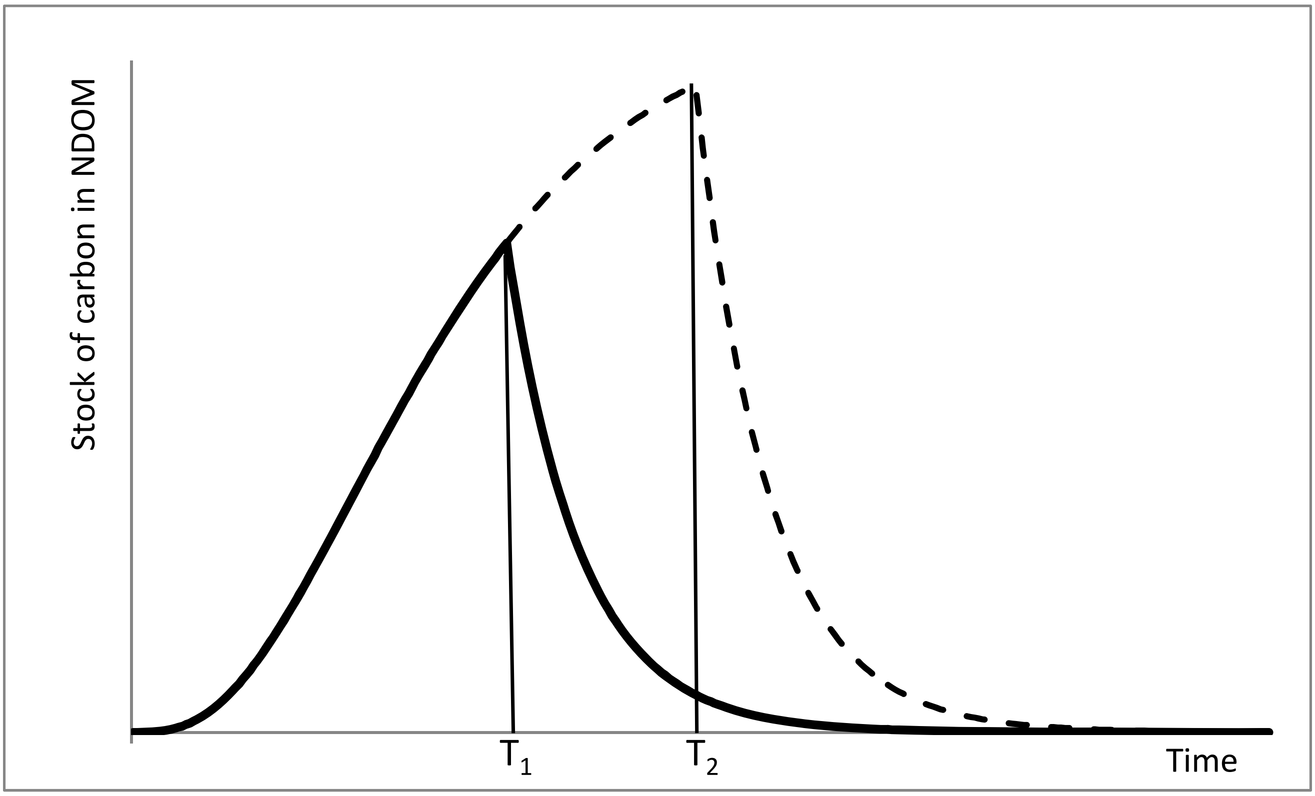


Figure 3. **The figure shows the development of the stock of NDOM generated after time t=0. The solid curve represents a case with harvest at time $t=T_1$, whereas the dashed curve represents the case when harvest takes place at time $t=T_2$.**



All parameter values are given in Table 1. In addition, it is assumed that the net commercial profit from harvest, before subtraction of the fixed costs, is 41.85 CAD/m^3, as was assumed in Asante and Armstrong (2012). (As each m^3 of wood is assumed to contain 0.2 tC, the commercial profit is 209.3 CAD/tC or 57,1 CAD/tCO_2 if the wood's carbon content is used as the unit.)

Results

The results are described in Table 2 and Fig. 2. Consider first Table 2. The first line with numbers shows the optimal harvest age if it is assumed that there are no social costs related to release of carbon to the atmosphere. This gives an optimal harvest age of 69 years.

Table 2 also shows how optimal harvest age changes if a social cost of carbon emissions is taken into account. We have combined a set of different carbon prices with different sets of carbon pools.

The second column shows the estimated optimal harvest age when only the carbon pool of the stems is taken into account. This is the approach taken by van Kooten et al. (1995), except that they considered multiple rotations and had an infinite time perspective. As in van Kooten et al. (1995), we find that the higher is the social cost of carbon emissions, the higher is the harvest age. Moreover, if the social cost of carbon exceeds a certain threshold, the forest should not be harvested; see the dotted curve in Fig. 2.

The third column shows the estimated optimal harvest age when it is also taken into account that other living biomass, such as roots, stumps, tops, and branches, represents important carbon capture in the growth phase. Here, we also take into account the fact that, after harvest, these parts of the trees are considered to be residues that gradually decompose and release carbon. The result is still that the higher is the social cost of carbon, the higher is the optimal harvest age, and that if the social cost of carbon exceeds a certain threshold, the forest should not be harvested. If we compare this case with the case where only the carbon of the stems was taken into account, then the harvest age is higher and the threshold value of the social cost of carbon, above which the forest should not be harvested, is lower; see also Fig. 2.

The fourth column of numbers in Table 2 shows the case when carbon capture and release from NDOM is taken into account. Inclusion of NDOM results in an even *higher* harvest age than in the case where NDOM was not taken into account. Fig. 3 provides some intuition behind this result. It illustrates that if harvest is delayed from time T_1 to time T_2, then the stock of NDOM is higher over the entire time span $t \in (T_1, \infty)$ than it would have been in the case with harvest at time T_1.

This result is in contrast to the results in both Asante et al. (2011) and Asante and Armstrong (2012). They found that incorporating DOM had the effect of reducing the harvest age.[1] Their result at this point must be seen in the light of

[1] It should be noted here that we did not carry out model simulations to include living biomass

their limited time perspective. Release of carbon from decomposition of residues and NDOM after the time of harvest is not included in their model.

The last column in Table 2 presents results that take into account the fact that, at the time of harvest, a share of the harvest is transferred to the wood products pool, and the release of carbon from this pool after the time of harvest. We find that this extension of the analysis leads to shorter rotation periods. The intuitive explanation of this result is simply that, when it is taken into account that a share of the wood products is not combusted immediately after harvest, but stored in buildings, etc., then the social cost of harvest is reduced and the optimal harvest age thus moves in the direction of what is commercially optimal, a result which is in agreement with Asante et al. (2011), Asante and Armstrong (2012), van Kooten et al. (1995), and Hoel et al. (2012).

Conclusion

Asante et al. (2011) and Asante and Armstrong (2012) pointed to the importance of including DOM and the accumulation of carbon in wood products when optimal harvest management is analysed. This note has confirmed that they pointed to important aspects of the analysis of forest management. Nevertheless, some of their conclusions result from their limited time scale. One issue is their consideration of a single rotation period only. This could be defended as an appropriate simplification, although we prefer the multiple rotation approach, which we took in Hoel et al. (2012). However, the problem is that both Asante et al. (2011) and Asante and Armstrong (2012) did not take account of the release of carbon from NDOM, residues, and the wood product pool after the time of the considered harvest. In this paper, we have shown that, when these parts of a more consistent model are included, the conclusions change significantly. Their finding that the initial stocks of DOM and wood products influence optimal harvest age negatively does not hold. The initial stocks of DOM and wood products do not matter with regard to optimal harvest age. Asante et al. (2011) and Asante and Armstrong (2012) also found that inclusion of DOM leads to a lower optimal harvest age. Our numerical simulations indicate that this conclusion is turned around when an infinite time perspective is taken. It is then found that inclusion of naturally dead organic matter leads to a *higher* optimal harvest age.

other than stems, but exclude residues, as Asante and Armstrong (2012) did. We think that this type of simulation would represent an incomplete analysis as it means that living biomass other than stems as a carbon pool simply disappears at the time of harvest.

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