Price and welfare effects of emission quota allocation

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Abstract:
We analyze how different ways of allocating emission quotas may influence the electricity market. Using a large-scale numerical model of the Western European energy market, we show that different allocation mechanisms can have very different effects on the electricity market, even if the total emission target is fixed. This is particularly the case if output-based allocation (OBA) of quotas is used. Gas power production is then substantially higher than if quotas are grandfathered or auctioned, and the price of emissions is almost twice as high. Moreover, even though electricity prices are lower, the welfare costs of attaining a fixed emission target are significantly higher. The paper analyzes other allocation mechanisms as well, leading to yet more outcomes in the electricity market. The numerical results for OBA are supported by theoretical analysis, with some new general results.

Keywords: Quota market; Electricity market; Allocation of quotas

JEL classification: D61; H23; Q41; Q58

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1 Introduction

Emissions trading has become the most important policy instrument for regulating emissions to air, with the EU Emissions Trading System (EU ETS) being the most prominent example so far.\(^1\) An important element in any emission trading system (ETS) is how to allocate allowances, i.e., the rights to emit.

Traditionally, economic analyses have focused on the comparison between auctioning and (unconditional) grandfathering, pointing particularly to the following: First, both alternatives provide a cost-effective outcome (Montgomery, 1972). Second, auctioning is generally preferred over grandfathering from an economic welfare perspective due to auction revenues that can be used to reduce distortionary taxes (Goulder, 1995). More recent literature has investigated the impacts of alternative allocation mechanisms, not least because of the various allocation rules applied and proposed in the EU ETS (see, e.g., Böhringer and Lange, 2005a,b; Fischer and Fox, 2007; Ellerman, 2008; Rosendahl, 2008; and Rosendahl and Storøsten, 2011a). One main insight from this mainly analytical literature is that the choice of allocation mechanism not only has distributional impacts, but can also affect prices and quantities in the markets regulated by the ETS, and thus the cost-effectiveness of the system.

In this paper we explore how various allocation mechanisms may play out in the electricity market, using the European market as our context. Most of the literature on allocation mechanisms so far have either had an analytical perspective (e.g., the cited references above), or used rather stylized numerical models (see below for some exceptions). Our paper is, as far as we know, the most in-depth numerical analysis of this issue so far. We find that the choice of allocation mechanism can indeed have substantial impacts on electricity prices and market shares, even if the overall emission target is held fixed. Moreover, the costs of complying with an emission target may increase substantially if certain allocation mechanisms are chosen.

In particular, our numerical results suggest that allocating quotas in proportion to quantities determined by the firms themselves, such as production or installed capacity, may induce large changes in prices, market shares and welfare costs. This is especially true for output-based allocation (OBA), for which allowances are distributed in proportion to each producer’s output, using so-called

\(^1\) http://ec.europa.eu/clima/policies/ets/index_en.htm
allocation factors. In this case we find that the welfare costs of complying with a certain emissions target may increase by 70 percent relative to grandfathering (or full auctioning). This scenario is characterized by a much higher price of emissions and a considerable increase in the market share of gas power (relative to grandfathering).

The model we apply is an extensive numerical equilibrium model of the Western European energy market (LIBEMOD, cf. Aune et al. 2008). The electricity market interacts in the model with the markets of other energy goods, allowing us, for example, to calculate the welfare costs of different scenarios. Electricity production and transmission between countries, including investments in new production and transmission capacity, are modeled in detail, and electricity consumption is endogenously determined by the price of electricity and other energy carriers. For the purpose of the current study, the model has been extended to incorporate alternative allocation mechanisms. We describe the model in more detail in Section 3.

In order to better understand the numerical findings with regards to OBA, we first use a simple analytical model to show some general results about the effects of this allocation mechanism relative to grandfathering. We first demonstrate that OBA generally leads to a higher price of emissions, a result which is well known from earlier studies, e.g., Fischer (2001) and Fischer and Fox (2007). As opposed to those two studies, we consider heterogeneous firms rather than a representative firm. Higher emission price is clearly most detrimental to the most emission-intensive plants, and we show that less emission-intensive plants will in general be better off under OBA. Moreover, we find that more fuel efficient plants will benefit from OBA, but only if non-fuel costs are strictly convex. If non-fuel costs are linear, however, output is unchanged if plants only differ with respect to fuel efficiency.

One interesting insight from the analytical part is that both demand for and supply of quotas are declining in the price of quotas under OBA, given a fixed set of allocation parameters and the government’s net sales of quotas (e.g., through auctioning): a higher emission price leads to lower emissions (thus lower demand) and lower output (thus lower supply through OBA). This has bearings

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2 Variants of OBA will be used in the next phase of the EU ETS (see Section 4 for details), and have also been included in proposed cap-and-trade systems in the U.S. (e.g., House of Representatives, 2009). Whereas the EU ETS will allocate quotas based on past output, most studies of OBA (including ours) assume that quotas are allocated based on current output. OBA has previously been analyzed numerically in both electricity market models (see below) and CGE analysis (Böhhringer and Lange, 2005a; Fischer and Fox, 2007; Böhhringer et al., 2010). Analytical studies (e.g., Fischer, 2001; Böhhringer and Lange, 2005b; Sterner and Muller, 2008; Rosendahl and Storrøsten, 2011b) have focused on e.g. price and quantity effects, first- and second-best allocation mechanisms, and clean technology investments. The former study is the most relevant here, and is further discussed below.
on the government’s supply-side policy in the quota market. If the government adjusts the OBA allocation parameter, i.e., changes the number of allowances per unit output, then it must also adjust either its net sales of quotas or the lump sum allocation of quotas to keep up with a fixed overall emissions target. Hence, demand side policy must go along with a suitable supply side management. We believe this point has not been adequately stressed in previous OBA studies.

Our paper relates to different strands of the literature, and we have already pointed to some related analytical studies on the effects of OBA. As mentioned above, only a few numerical studies exist on the relationship between allocation mechanisms and the electricity market. One such study is Neuhoff et al. (2006). They use power dispatch models for the UK and the European electricity market, with fixed demand for electricity, to examine how different allocation rules may affect CO₂ emissions, prices of CO₂ and electricity, and the generation mix. Burtraw et al. (2006) investigate the impacts of different allocation mechanisms in the Regional Greenhouse Gas Initiative (RGGI) among North Eastern states in the U.S., using a simulation model of the electricity market in those states. Bode (2006) simulates the effects of alternative allocation schemes using a power market model with five different technologies and 110 power plants (located in a number of countries in different parts of the world). These three studies cover some of the same issues as we focus on, but their models are not as rich as the LIBEMOD model, e.g., with respect to inter-fuel competition, investments in different types of capacity and welfare calculations. Their main findings with respect to prices and quantities (e.g., under OBA) are largely consistent with the results we obtain.

Another strand of literature investigates the relationship between emissions trading on the one hand, and electricity prices and quantities on the other, either from a general perspective or in the EU ETS context. Yet, this literature has typically overlooked the effects of different allocation mechanisms. A large part of this literature has examined to what degree CO₂ prices are passed on to electricity prices.. Some studies have estimated the elasticity of the electricity price with respect to the CO₂ price, finding that this elasticity strongly depends on the generation mix in the market.³ Other studies have estimated the marginal pass-through rate, i.e., the ratio of the increase in power price to the marginal generator’s CO₂ cost. For example, Lise et al. (2010), using a short-run simulation model covering the electricity market of 20 European countries⁴, find that the marginal pass-through rate is 70-90 percent in most countries and scenarios considered. Econometric studies find similar pass-through rates.⁵ In

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³ For instance, Bunn and Fezzi (2007) for the UK, and Kirat and Ahamada (2011) for Germany and France.
⁴ See Chen et al. (2008) for an earlier version of the same model.
⁵ For instance, Sijm et al. (2006) for Germany and the Netherlands, Simshauser and Doan (2009) for Australia, and Fell (2010) for the Nordic market.
comparison, our equilibrium pass-through rate (with unconditional allocation of quotas) is actually slightly above 100 percent. This occurs because of higher gas prices due to increased gas demand. More importantly, with alternative allocation mechanisms, the pass-through rate tends to be lower, substantially so (at around one third) under output-based allocation.

In the next section we derive some analytical results about the effects of output-based allocation. Then in Section 3 we describe the numerical model LIBEMOD. Section 4 lays out the various policy scenarios we consider and compare these with previous, current and upcoming allocation rules in the EU ETS. Note that although our numerical analysis is performed in a European context, the allocation mechanisms investigated are not a blueprint of the numerous allocation rules applied in the EU ETS. Instead, our aim is to examine a few distinct and frequently used, or proposed, allocation mechanisms, focusing on one alternative at a time. Section 5 presents the numerical results on how different allocation mechanisms may affect the electricity market, and Section 6 concludes.

2 General effects of output-based allocation

2.1 Producer behaviour

In this section we use a theoretical model to study output-based allocation schemes. We consider a two sector economy where in each sector \( s, s = 1,2 \), there is production of a homogenous good. Let \( y_{is} \) be production in firm \( i \) in sector \( s \). Production requires use of fossil fuels \( x \) (measured in toe); let \( y_{is} = y_{is}(x_{is}) \) be the production function of producer \( i \) in sector \( s \), which is increasing and concave. The input requirement function (inverse function of the production function), \( x_{is} = x_{is}(y_{is}) \), is then increasing and convex, and shows how much fossil fuels \( x_{is} \) that is required to produce \( y_{is} \).

Use of fossil fuels generates \( \omega_{is}x_{is}(y_{is}) \) units of CO\(_2\) emissions where \( \omega_{is} > 0 \) is the emission coefficient of firm \( i \) in sector \( s \). Let \( \kappa_{is} \) be the price of emissions in sector \( s \). We assume that in sector 1 emissions are regulated through tradable quotas, whereas in sector 2 an emission tax \( \kappa_{is} \) is imposed.

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6 Which power producing technology is on the margin will vary across time and across countries (both in reality and in our model). In general the demand for all fuels will change if electricity prices and/or CO\(_2\) prices change, and the equilibrium effects in all markets simultaneously will determine the actual pass-through rates. The effect of changes in the markets for natural gas tends to be the most important since the supply of natural gas is much less elastic than the supply of e.g. coal.
In sector 1 producer \(i\) is required to have \(\omega_i x_{1i}(y_{1i})\) units of CO\(_2\) quotas in order to produce \(y_{1i}\). Further, for a moment we assume that in sector 1 producer \(i\) receives (free) quotas from one source only, namely \(y^0_{1i}\) tradable quotas from the government. Here, \(y^0\) is a parameter (decided by the government) and \(y^0_{1i}\) is a pre-determined variable, here production in firm \(i\) in sector 1 in an earlier period. Hence, the net cost of purchasing quotas is \(\kappa_i(\omega_i x_{1i}(y_{1i})-y^0_{1i})\).

Finally, let \(p_s\) be the output price in sector \(s\), and let \(c_{is}(y_{is})\) be the costs of producing \(y_{is}\), which is assumed to be increasing and convex, see discussion below. Note that \(c_{is}(y_{is})\) covers all costs associated with producing \(y_{is}\) except costs of emissions.

Producer \(i\) in sector 1 maximizes \(p_i y_{1i} - c_i(y_{1i}) - \kappa_i(\omega_i x_{1i}(y_{1i})-y^0_{1i})\) wrt. \(y_{1i}\), which yields the well-known first-order condition

\[
p_1 = c_{1i}'(y_{1i}) + \kappa_i(\omega_i x_{1i}'(y_{1i})), \forall i. \tag{1}
\]

According to (1), at the margin the benefit of a marginal increase in production, \(p_1\), should equal the corresponding marginal cost, \(c_{1i}'(y_{1i}) + \kappa_i(\omega_i x_{1i}'(y_{1i}))\), where the second term is the net marginal emission cost of a unit of production. Note that \(y^0_{1i}\) is not contained in (1), which simply reflects that this variable is predetermined. Hence, an increase in \(y^0\) will have no effect on the decision of the firm – it will only imply a lump-sum transfer of money from the government to the firm.\(^7\)

Assume, alternatively, that the firm receives quotas not only related to its past activity \((y^0_{1i})\), but also related to its current activity such as current production, \(y_{1i}\) (output-based allocation). The number of quotas received (in addition to \(y^0_{1i}\)) is \(\gamma y_{1i}\), where \(\gamma\) is a parameter decided by the government, common to all firms in sector 1. In this case the firm maximizes \(p_i y_{1i} - c_i(y_{1i}) - \kappa_i(\omega_i x_{1i}(y_{1i})-\gamma y_{1i}-y^0_{1i})\) wrt. \(y_{1i}\), which yields the first-order condition

\[
p_1 = c_{1i}'(y_{1i}) + \kappa_i(\omega_i x_{1i}'(y_{1i})-\gamma), \forall i. \tag{2}
\]

\(^7\) If producers think that future allocation may depend on current production, i.e., so-called updating (Rosendahl, 2008), the first-order condition will change. This possibility is not considered in our analysis.
In (2) the firm takes into account that increased production will lead to increased costs 
\( c'_i(y_{i1}) + \kappa_i \omega_i x'_i(y_{i1}) \) but also to more quotas received for free \(-\kappa_i \gamma_i\). Hence, the net marginal emission cost is now “only” \( \kappa_i (\omega_i x'_i(y_{i1}) - \gamma_i) \), which is lower than in the previous case, \( \kappa_i \omega_i x'_i(y_{i1}) \), see (1). In order to make the problem interesting and relevant, we assume \( \omega_i x'_i(y_{i1}) - \gamma_i > 0 \) for all firms, that is, net marginal emission cost is positive. An increase in \( \gamma \) will have impact on the decision of the firm; marginal emission costs will decrease, and hence the firm will increase its production and emissions.

In the other sector, referred to as the tax sector \((s = 2)\), there is a tax \( \kappa_2 \) imposed on all emissions. Each firm \( i \) maximizes \( p_2 y_{i2} - c_{i2}(y_{i2}) - \kappa_2 \omega_{i2} x_{i2}(y_{i2}) \), and the corresponding first-order condition is

\[ p_2 = c'_{i2}(y_{i2}) + \kappa_2 \omega_{i2} x'_{i2}(y_{i2}), \forall i, \]

which is equivalent to (1).

Above we have discussed the decisions of a firm. Clearly, changes at the firm level have market implications; if firms receive quotas according to their level of production \((\gamma_i y_{i1})\), and \( \gamma \) increases, the price of quotas may change. This will lead firms to adjust inputs and outputs. Therefore, we now consider equilibrium effects of shifts in output-base allocation rules.

### 2.2 Market equilibrium

Below we distinguish between two policy cases; Case A where emissions in each sector are fixed (and the price of emissions typically differs across sectors), and Case B where the price of emission is the same in the two sectors. In both cases, total emissions in the economy are fixed.

In each sector, firms produce a homogenous good and the price of the good depends on total production of the good represented by the inverse demand function for good \( s \):

\[ p_s = p_s(\sum y_{is}) \quad p'_s \leq 0. \]

The first-order condition for firms is given by relations (2) and (3), which for given levels of \( p_s, \kappa_s \) determines the supply of the goods \( y_{is} \). Further, total demand for emissions in each sector is given by
\[ D_s = \sum_{i} \alpha_{si} x_i(y_{si}), \]  

(5)

which are functions of \( p_s, \kappa_s \) through \( y_{si} \).

In the quota sector, demand for quotas equals the demand for emissions, while the total supply of quotas is:

\[ T_i = \sum_{i} (\gamma y_{si} + \gamma^0 y^0_{si}) + g \]  

(6)

where \( \sum_{i} (\gamma y_{si} + \gamma^0 y^0_{si}) \) are quotas received by firms and \( g \) is net sales of quotas from the government. Thus \( T_i \) is also a function of \( p_s, \kappa_i \). Equilibrium in the quota market is given by:

\[ D_i = T_i \]  

(7)

Actual emissions in each sector, \( e_s \), must equal total demand for emissions:

\[ e_s = D_s. \]  

(8)

Finally, total emissions in the economy, \( E \), equal the sum of emissions in each sector:

\[ E = e_1 + e_2. \]  

(9)

**Case A: Fixed sectoral emissions**

Below we will always treat \( E \) as an exogenous variable. With fixed sectoral emissions, both \( e_1 \) and \( e_2 \) are de facto exogenous variables, but formally one of them will be determined from relation (9), and hence endogenous. Below we let \( e_2 \) be endogenous.

Relations (2) - (9) determine the variables \( y_{si}, p_s, \kappa_s, D_s, T_s, \gamma^0 \) and \( e_2 \), given the exogenous variables \( \gamma, g, E \) and \( e_1 \). Note that \( g \) may take “any” value (provided there is a corresponding equilibrium), for example, zero. Furthermore, we may alternatively let \( g \) or \( \gamma \) be an endogenous variable and \( \gamma^0 \) an exogenous variable. Also note that there is no interaction between the two sectors: Relations (2), (4) (for \( s = 1 \)), (5) (for \( s = 1 \)), (6), (7) and (8) (for \( s = 1 \)) determine \( y_{si}, p_s, \kappa_s, D_s, T_s \) and \( \gamma^0 \) given the exogenous variables \( \gamma, g \) and \( e_1 \), and relations (3), (4) (for \( s = 2 \)), (5) (for
Case B: A common price of emissions

Now there is a common price of emissions, that is,

\[ \kappa_1 = \kappa_2 = \kappa. \]  

(10)

Because of (10) the two sectors are now interlinked. The fourteen relations (2) - (10) determine the variables \( y_i, p_s, \kappa_s, \kappa, D_s, T_s, e_s \) and \( y^0 \), given the exogenous variables \( \gamma, g \) and \( E \).

2.3 Equilibrium effects of changes in the allocation parameter

We now examine how a change in \( \gamma \), i.e., the output-based allocation parameter, affects the equilibrium. We start with the case of fixed sectoral emissions, and then examine the case of a common price of emissions.

Case A: Fixed sectoral emissions

From relations (2) and (4) we have

\[ y_{it} = \hat{y}_{it}(\kappa_i, \gamma). \]  

(11)

Differentiation of (2) with respect to \( \gamma \), using (11), gives \( \frac{\partial \hat{y}_{it}}{\partial \gamma} > 0 \). Moreover, by differentiating (2) with respect to \( \kappa_i \) and using (11), we can derive an expression for \( \frac{\partial \hat{y}_{it}}{\partial \kappa_i} \). Typically, \( \frac{\partial \hat{y}_{it}}{\partial \kappa_i} \) is negative, but if production units are “sufficiently different”, this partial derivative may be positive for some units. For example, units with a very low emission coefficient (\( \omega \)), and/or firms requiring only a small
increase in the use of fossil fuels for a given increase in production \((x'(y))\), are candidates for a positive derivate.\(^8\) Below we assume that \(\frac{\partial y_{i1}}{\partial \kappa_i} < 0\), for all firms.

Inserting (11) into (5) and (6), and differentiating with respect to \(\kappa_i\), we find how aggregate demand for quotas, and aggregate supply of quotas, depend on the price of quotas. Because \(\frac{\partial y_{i1}}{\partial \kappa_i} < 0\), both demand for, and supply of, quotas are decreasing in the price of quotas. In Figure 1 we have depicted demand for, and supply of, quotas (for given values of \(g, \gamma^0\) and \(\gamma\)). In the figure the demand curve cuts through the supply curve from below. This requires that \(\sum \omega_i x_i' \frac{\partial y_{i1}}{\partial \kappa_i} < \sum \gamma \frac{\partial y_{i1}}{\partial \kappa_i}\), that is, \(\sum (\omega_i x_i' - \gamma) \frac{\partial y_{i1}}{\partial \kappa_i} < 0\), which is fulfilled under our assumption, reflecting that net marginal cost of emissions is positive \((\omega_i x_i'(y_i) - \gamma > 0)\).

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\(8\) With two firms in sector 1, here termed 1 and 2, then for firm 1 the derivative of production wrt. the quota price partly depends on the term \((\omega_1 x_1'(y_{i1}) - \omega_1 x_1'(y_{i2})) p_1\), whereas the corresponding term for firm 2 is \((\omega_2 x_2'(y_{i2}) - \omega_2 x_2'(y_{i1})) p_2\). We see that for firm 1 the term will be positive if \(\omega_1 x_1'(y_{i1}) < \omega_1 x_1'(y_{i2})\), and could possibly dominate the negative terms in the expression for \(\frac{\partial y_{i1}}{\partial \kappa_i}\).
In Figure 1, $D_1^0$ is the initial demand curve for quotas (emissions), $T_1^0$ is the initial supply curve of quotas, and $A$ is the initial equilibrium. Assume now that $\gamma$ increases. Under our assumptions, both demand for quotas and supply of quotas will shift upwards ($\frac{\partial \gamma_i}{\partial y_i} > 0$). The shift in aggregate demand (from $D_1^0$ to $D_1^1$), together with the assumption of a fixed level of emissions $\bar{e}_t$, determine the new equilibrium $B$, see Figure 1. As seen from the figure, the price of quotas $\kappa_i$ has increased.

We now examine in more detail how the equilibrium changes due to a shift in $\gamma$. First, the demand curve shifts by $\sum \omega_i x_{it} \frac{\partial \hat{y}_{it}}{\partial y}$, whereas the supply curve shifts by $\sum (\gamma \frac{\partial \hat{y}_{it}}{\partial y} + y_{it})$ (In Figure 1, $T_1^1$ is the supply curve after the shift in $\gamma$). Under our assumptions we do not know whether the shift (measured vertically) is larger for demand than for supply. In Figure 1 we have assumed that the shift in demand is smallest, which is consistent with the numerical cases in Section 5. Then the government has to decrease $\gamma^0$ by so much that the new supply curve ($T_1^2$ in Figure 1) goes through $B$.

So far we have assumed that the government’s sale of quotas is given. Assume the opposite, that is, the government sells or buys quotas as a response to market changes, and keeps $\gamma^0$ fixed. Then $g$ is an endogenous variable, and both $\gamma^0$ and $\gamma$ are exogenous. If now $\gamma$ increases, then again $D_1^0$ shifts to $D_1^1$, and hence $B$ still represents the new equilibrium. Thus, all prices and quantities are identical between these two cases. However, the government now has to adjust $g$ such that $T_1^2$ goes through $B$. Hence, in Figure 1 $g$ is decreased (instead of $\gamma^0$). From (6) we know that the change in $g$, when $\gamma^0$ is held fixed, has to equal the change in $\sum \gamma^0 y_{it}^0$, if $g$ had been held fixed.

The discussion above can be summarized as follows:

**Proposition 1:** If sectoral emissions are given, and firms in the quota sector receive more quotas for each unit of production ($\gamma$ increases), the price of emissions increases. To reach the new equilibrium the government has to either change the number of free quotas related to firms’ emissions in an earlier period (adjust $\gamma^0$), or change its net sale of quotas (adjust $g$).

As mentioned in the introduction, the first part of Proposition 1 has been shown in previous studies such as Fischer (2001) and Fischer and Fox (2007). Note, however, that we assume heterogeneous
firms, each with a concave production function. This is in contrast to the two above-mentioned studies, which assumed homogenous firms (modelled as a representative firm), whose unit costs are decreasing in their emissions rate but otherwise constant.

If firms are identical, an increase in $\gamma$ will not change emissions at the firm level simply because total emissions are given in the quota sector. Hence, also production at the firm level does not change, and therefore the output price (in the quota sector) is unaltered.

With heterogeneous firms, an increase in $\gamma$ will affect emissions and production at the firm level, and therefore also market shares will change. It seems reasonable to presume that firms that are fossil-fuel intensive, or firms with a high emission coefficient, will, cet. par., decrease their market shares, reflecting that these firms will suffer the most from a higher price of quotas (which is caused by the increase in $\gamma$). In Appendix A we show that in general these conjectures are correct. We assume that costs of production consist of two terms, fuel costs and non-fuel costs (the latter is identical across firms), and prove the following propositions:

**Proposition 2:** Suppose that sectoral emissions are given and that there are two types of firms in the quota sector; type 1 is more fossil-fuel intensive than type 2. Assume that firms receive more quotas for each unit of production ($\gamma$ increases). If non-fuel costs are linear, then production in each firm does not change. If non-fuel costs are strictly convex, then production in type 1 (2) firm decreases (increases) and total production increases.

**Proposition 3:** Suppose that sectoral emissions are given and that there are two types of firms that differ only with respect to the emission coefficient. If firms receive more quotas for each unit of production ($\gamma$ increases), then production in firms with the lowest (highest) emission coefficient increases (decreases), and total production increases.

The intuition behind Propositions 2 and 3 is straightforward. If firms receive more quotas for each unit of production, then demand for quotas increases and total production tends to increase. Because total emissions in the sector are fixed, the price of quotas increases. All firms are hurt by the raise in the quota price, but firms that are fossil-fuel intensive, or have high emission coefficient, are hurt the most. In these firms production decreases. For the other type of firm, the initial effect of a lower $\gamma$, which decreases marginal cost of production, dominates the (generated) effect of a higher quota price, that is, in these firms production increases. Finally, in the special case of linear non-fuel costs and
identical emission coefficients (first part of Proposition 2), by construction the magnitude of the shifts do not differ between firms (see the Appendix), and hence production does not change.

Above we have examined the impact of an increase in \( \gamma \). Such an increase can also be seen as a shift in the regulatory system from the case of firms only receiving quotas according to their historical activity \( (\gamma^0 > 0, \gamma = 0) \), to the case of firms also receiving quotas according to their current activity \( (\gamma^0 > 0, \gamma > 0) \). We have shown that such a transition will increase the equilibrium price of quotas, and will in general have impact on production if firms are heterogeneous. We now turn to the case of a common price of emissions.

**Case B: A common price of emissions**

In Figure 2 we have depicted the equilibrium when the imposed emission tax is set equal to the quota price. Figure 2 is a bath tub diagram where the length of the diagram is equal to total emissions, \( E \). Emissions in sector 1, \( e_1 \), is measured from left to right, whereas emissions in sector 2, \( e_2 \), is measured from right to left. In the figure \( D_1^0 \) is the initial demand curve for quotas (emissions) in the quota sector, and \( T_1^0 \) is the initial supply curve for quotas, that is, the left side of Figure 2 is a copy of Figure 1. Further, \( D_2 \) is demand for emissions in the tax sector, measured from the right. Point \( A \) represents the initial equilibrium. Here, \( D_1^0 \) cuts through \( D_2 \) reflecting equal marginal willingness to
pay for increased emissions. Further, equilibrium emissions in sector 1 are $e_1^0$, and the common equilibrium price of emissions is $\kappa^0$.

Assume now that $\gamma$ increases. Like above, the demand curve in the quota sector shifts upwards (to $D_1^1$), whereas the demand curve in sector 2 does not shift. The new equilibrium is represented by $B$ where $D_1^1$ cuts through $D_2$. Like above, the supply curve in the quota sector will shift; first to $T_1^1$ because of the shift in $\gamma$ (not shown in Figure 2), and then to $T_1^2$. The latter shift is accomplished either through an adjustment of $\gamma^0$ (the number of free quotas related to firms’ emissions in an earlier period) or through an adjustment of $g$ (the government’s net sales of quotas). In equilibrium, the supply curve $T_1^2$ goes through $B$ in Figure 2. As seen from Figure 2, the common price of emissions has increased (from $\kappa^0$ to $\kappa^1$), as has the emissions in sector 1 (from $e_1^0$ to $e_1^1$), while emissions in the tax sector has decreased (from $e_2^0 = E - e_1^0$ to $e_2^1 = E - e_1^1$).

The discussion above can be summarized by the following proposition:

**Proposition 4:** If there is a common price of emissions in the two sectors and firms in the quota sector receive more quotas for each unit of production ($\gamma$ increases), the government has to either change the number of free quotas related to firms’ emissions in an earlier period (adjust $\gamma^0$), or change its net sale of quotas (adjust $g$). The price of emissions increases, emissions in the quota sector increase, whereas emissions in the tax sector decrease.

From Proposition 4 we know that use of fossil fuels decreases in the tax sector, whereas the opposite is the case in the quota sector. Hence, production in the tax sector will for sure decrease, whereas in the quota sector production at the firm level will increase if firms are identical. The case of heterogeneous firms is examined in Appendix A. There we show that:

**Proposition 5:** Suppose there is a common price of emissions in the two sectors and firms in the quota sector receive more quotas for each unit of production ($\gamma$ increases). If there are two types of firms in the quota sector and these differ only with respect to the degree of fossil-fuel intensity, then the least fossil-fuel intensive firms increase their production, total production in the quota sector increases, whereas the impact on production for the other type of firm is ambiguous.
Proposition 6: Suppose there is a common price of emissions in the two sectors and firms in the quota sector receive more quotas for each unit of production ($\gamma$ increases). If there are two types of firms in the quota sector and these differ only with respect to the emission coefficient, then firms with the lowest emission coefficient increase their production, total production in the quota sector increases, whereas the impact on production for the other type of firm is ambiguous.

The results in Propositions 5 and 6 are quite similar to those in Propositions 2 and 3. The exception is the impact on firms being fossil-fuel intensive or having a high emission coefficient. In Propositions 2 and 3, production in these firms decreases (or is unchanged in a special case), whereas in Propositions 5 and 6 the effect is ambiguous. The difference reflects that in the latter case emissions decrease in sector 2, thereby opening up for increased emissions in sector 1 because total emissions are fixed.

3 Description of the numerical model LIBEMOD

LIBEMOD is a numerical multi-market equilibrium model for the energy sector. Its main focus is on the electricity and natural gas industry in Western Europe, but it also covers markets for other fuels like coal and oil. The model is a synthesis of the bottom up and top-down modeling traditions. On the one hand it offers a detailed description of the electricity and natural gas industry in Western Europe, in particular production of electricity, and on the other hand it has a clear foundation in economic theory by deriving structural behavioral relations from well-specified optimization problems, and imposing that all markets should clear.

LIBEMOD is well suited to analyze the impact of different allocation mechanisms for CO$_2$ quotas because the model has a rich description of costs in electricity production. In the model, the responses of profit-maximizing electricity producers to allocation mechanisms have impacts on production and investment of electricity, and might also differ between electricity technologies, for example between coal-fired plants, gas-fired plants and renewables. LIBEMOD determines all energy quantities and all energy prices, and CO$_2$ prices are also determined within the model.

We now give a more detailed description of the model; see Aune et al. (2008) for a complete description of the model, including detailed documentation of data sources and calibration strategy. LIBEMOD distinguishes between model countries and other countries. In a model country, there is production, investment, trade and consumption of all energy goods, that is, electricity, natural gas, three types of coal (coking coal, steam coal and lignite), oil, and biomass. In LIBEMOD, each of sixteen countries in Western Europe (Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Spain, Sweden, Switzerland, Portugal and the United
Kingdom) is a model country, whereas all other countries in the world are represented mainly through supply of, and demand for, coal and oil.

Electricity and natural gas are traded in competitive well-integrated Western European markets, using existing capacity in international transmission of these energy goods. These capacities will be expanded (in the model) if there are profitable investment opportunities. Coking coal, steam coal and oil are traded in competitive global markets, whereas lignite (used for emission-intensive coal-power production) and biomass (used to produce electricity) is traded in national markets only.

There are four groups of users of energy in each model country. First, there is intermediate demand from electricity producers: for example, gas power producers demand natural gas. Furthermore, there is demand from end users: the household, industry and transport sectors, though the latter demands only oil products. For end users, demand is derived from a nested CES utility function with five levels. At the top-nest level, there are substitution possibilities between energy-related goods and other forms of consumption. At the second level, consumers face a trade-off between consumption based on the different energy sources. Each of these is a nest describing complementarity between the actual energy source and consumption goods that use this energy source (for example, electricity and electrical appliances). Finally, the fourth and fifth levels are specific to electricity in defining the substitution possibilities between summer and winter (season) and between day and night. Thus, except for electricity, energy goods are traded in annual markets. Note that the calibrated parameters of the utility functions differ between end users and countries.

In each model country, there is production of electricity by various technologies: steam coal power, lignite power, gas power, oil power, reservoir hydro power, pumped storage power, nuclear power, waste power and wind power. Some of these are not available in all countries. There are costs related to electricity production: fuel costs, non-fuel operating costs, maintenance costs (related to the maintained power capacity), start-up costs and investment costs. The power producer obtains revenues either from using the maintained power capacity to produce and sell electricity, or by selling part of the maintained power capacity to a national system operator who buys reserve power capacity in order to ensure (if necessary) that the national electricity system does not break down.

Power producers face some technical constraints. For example, maintained capacity should not exceed installed capacity. In addition, there are technology-specific constraints. For example, for reservoir hydro, the reservoir filling at the end of a season cannot exceed the reservoir capacity, and total use of water cannot exceed total availability of water (the sum of seasonal inflow of water and reservoir filling at the end of the previous season). Each power producer maximizes profits subject to the
technical constraints. This optimization problem implies a number of first-order conditions, which determine the operating and investment decisions of the producer.

LIBEMOD distinguishes between existing power plants - those that were available for production in the data year 2000 - and new plants. For existing plants, the capacity is exogenously depreciated over time and cannot be expanded. Moreover, for each type of fossil fuel based technology, and for each model country, efficiency typically varies across existing plants. New plants are “built” in the model if such investments are profitable. For new fossil fuel based technologies, efficiency does not differ between the same type of plant, but differs across technologies.

LIBEMOD determines all energy quantities – investment, production, trade and consumption – all energy prices, both producer prices and end-user prices, and also emission of CO$_2$ by sector and country. The model has been calibrated to the year 2000, imposing that the parameters should reproduce observed demand, costs and efficiency distributions in 2000. For markets that we assume were competitive in 2000, that is, the coal and crude oil markets, calibrated prices will be identical to observed market prices. For other good, for example, natural gas and electricity, observed prices differ from calibrated prices, reflecting market imperfections in 2000 whereas we impose competitive markets in the calibration equilibrium.9

For the CES utility functions (one for each type of end-user in each model country), the parameters are calibrated to minimize the deviation from exogenous own-price and cross-price demand elasticities, where the target values are based on literature estimates. For households, the own-price elasticities are in the range of -0.4 to -0.6, while for industry the range runs from -0.5 to -0.7. For each model country there is a load curve with four segments – one for each time period. According to our data, demand is typically higher in winter than in summer (heating requires more energy than cooling), and higher during the day than at night. For a more detailed description of LIBEMOD, including data sources, see Aune et al. (2008).

4 Policy scenarios

In the numerical analysis we will examine the effects in the Western European energy market of different allocation mechanisms. The four sectors in LIBEMOD are divided into two groups: The electricity sector and the “rest-sector” (i.e., industry, household and transport sectors). For the former

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9 The model is solved in USD$_{2000}$ prices, but the model results are converted to Euro$_{2010}$ using a PPP conversion rate and a producer price index for the Euro area (http://stats.oecd.org/Index.aspx?DataSetCode=SNA_TABLE4)
sector we assume that all electricity producers in Western Europe are part of a common emission trading system (ETS). For the rest-sector we assume that each country implements a CO₂-tax, which is either country-specific or equal across countries (more details below). We will consider the year 2010, which is in the middle of the Kyoto period 2008-2012, and assume throughout that the Western European countries as a group comply with their Kyoto targets. Moreover, for each country, the sum of emissions in the rest-sector and the allocated quotas to the ETS must equal the country’s national emissions target. Note that in the policy scenarios we implicitly assume that the implemented policy was known to investors already in the year 2000, i.e., the data year of the model. This was obviously not the case – the results should therefore be interpreted as potential effects around ten years after the announcement of the different policies.

The description above has certain similarities to the actual policy situation in the EU, but there are also significant differences. In particular, although the electricity sector accounts for a majority of EU ETS emissions, the system also covers other energy producing sectors and most energy-intensive industries. Moreover, the EU ETS comprises all member states, not only the Western European ones. However, the objective of this study is not to mimic the EU ETS as such, but to get a better understanding of how different ways of allocating free quotas may have an influence on the electricity market.

We distinguish between two alternative sets of policy scenarios. The two sets correspond to the two cases analysed in Section 2 above, i.e., Case A and Case B (see Table 1). We assume throughout that there is no auctioning in the ETS, i.e., all issued quotas are allocated to the installations. For reasons of exposition we first present a set of scenarios corresponding to Case B, where we assume that the price of CO₂ is identical across countries and sectors. This requires that the common CO₂ tax for non-ETS sectors in all countries and the allocation factor for the ETS (i.e., electricity) sector are adjusted so that the ETS price equals the CO₂ tax and total emissions across countries and sectors equal the joint Kyoto target. With unconditional or lump sum allocation of quotas (or alternatively auctioning of quotas), this outcome will be cost-effective and equal to a scenario with a uniform CO₂-tax across all sectors and countries in Western Europe. If quotas are allocated differently, however, the outcome may be different. In particular, the share of emissions between the electricity sector and the rest-sector may depend on the chosen allocation mechanism.

10 As LIBEMOD does not model the energy markets of EU Member States beyond EU15, we consider the emission targets defined by the EU’s Burden Sharing Agreement (http://ec.europa.eu/clima/documentation/ets/docs/com_1999_230_en.pdf) for the EU countries.

11 This scenario will in general not be in accordance with the Burden sharing agreement mentioned above. However, the agreement can be attained by an appropriate level of quota trade between the governments, which would not affect the outcome of our partial equilibrium model.
It may be difficult for policy makers to fine-tune the allocation in the way described above. Moreover, some could argue that the share of emissions across sectors should be held fixed, so-called sector grandfathering. Thus, in the second set of policy scenarios corresponding to Case A in Section 2, we assume that the total allocation of quotas (and thus total emissions in the electricity sector in Western Europe) is fixed and based on the electricity sector’s share of total emissions in the base year of the model (i.e., 2000). Moreover, emissions in the rest-sector in each individual country are also fixed and derived from base year emissions. In these scenarios, the price of CO₂ will typically vary between sectors (and between countries for the rest-sector).¹² Note that total emissions over all model countries and sectors are the same in Case A and B.

<table>
<thead>
<tr>
<th>Policy scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case A</strong></td>
</tr>
<tr>
<td>Total emissions in the electricity sector is fixed(^a)</td>
</tr>
<tr>
<td>Country specific emissions in the rest-sector are fixed(^b)</td>
</tr>
<tr>
<td><strong>Case B</strong></td>
</tr>
<tr>
<td>Identical price of CO₂ across sectors and countries</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Allocation mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 “Unconditional grandfathering”</td>
</tr>
<tr>
<td>A2 “Conditional grandfathering”</td>
</tr>
<tr>
<td>A3 “Output-based allocation”</td>
</tr>
<tr>
<td>A4 “Capacity-based allocation”</td>
</tr>
</tbody>
</table>

\(^a\) The electricity sector’s share of total emissions is set equal to its share of total emissions in the base year.

\(^b\) The rest-sector’s share of a country’s emissions is set equal to its share in the base year.

In both Case A and B we consider four alternative allocation mechanisms, see Table 1. The benchmark mechanism A1 is unconditional grandfathering of quotas, i.e., a variant of lump sum allocation of quotas. The number of quotas is proportional to the level of emissions in the base year of the model (2000). LIBEMOD does not specify individual installations, but distinguishes between different electricity technologies, and between existing and new installations. Thus, the distribution of quotas is technology-specific and only goes to existing installations. Note that full auctioning of quotas would give the same outcome as A1 in LIBEMOD, except for the distribution of income between electricity producers and the government.

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¹² Alternatively, we could assume that the CO₂-price in the rest-sector is equalized across countries, e.g., realized through trade in quotas between member countries. The main results would not differ much from the ones in Case A though.
The second mechanism A2 also assumes that quotas are grandfathered to existing firms. However, quotas will be withdrawn from installations that shut down (i.e., do not maintain) their capacity. If, for example, maintained capacity of existing coal power producers in a country is reduced by $k$ percent compared to the base year level, coal power producers will only receive $(100-k)$ percent of the quotas they would have received if they did not shut down capacity. Obviously, this allocation mechanism gives increased incentives to maintain production capacity compared to the benchmark mechanism, as the producers lose valuable quotas if they shut down capacity.

The third mechanism A3 does not involve any grandfathering. Instead, quotas are distributed in proportion to current production, i.e., output-based allocation, which was examined in the analytical model in Section 2. As shown in Section 2, this allocation mechanism acts as an implicit production subsidy, and hence gives increased incentives to produce electricity. Note, however, that only fossil-based electricity production receives quotas, in line with e.g. the EU ETS.

Finally, the fourth mechanism A4 assumes that quotas are distributed in proportion to maintained capacity. Thus, similar to A3, mechanism A4 is connected to present activity, not to the past. This mechanism will give higher incentives to invest in and maintain production capacity, but not to produce (at least not directly). Again, only fossil-based power plants receive quotas.

Comparing with the allocation rules in the EU ETS, allocation in the first two periods (2005-2007 and 2008-2012) has varied across member states and sectors, but in general it has been a mixture of allocation mechanism A2 for existing installations and A4 for new installations and capacity expansions at existing installations. In the next period (2013-2020), allocation rules are mostly harmonized across the EU. Electricity producers will as a default no longer receive allowances, but ten of the new member states (EU-12 minus Slovakia and Slovenia) have got an option to allocate a limited number of allowances to electricity generators that have been in operation since (or had started to invest before) the end of 2008. The member states can then choose between allocation mechanisms similar to A2 or A4. Other sectors will still be given substantial amounts of free allowances (in all member states), using a mixture of allocation mechanisms similar to A3 and A4.

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13 Allocation rules have not been harmonized across EU member states, so the comparison is only an approximation. For instance, some countries have chosen different allocation parameters for investments in different power technologies, whereas others have chosen equal parameters. Note that combining A2 for existing installations and A4 for new installations (and capacity expansions) gives the same outcome as allocation mechanism A4 in our model simulations.

14 Allocation to an installation will be proportional to historic production level (2007-8), but adjusted whenever the operating capacity is reduced or increased substantially. New installations will receive quotas in proportion to operating capacity.
One interpretation of our results is therefore that they may provide insight into how the links between the EU ETS and the power market in Western Europe may change when going from the second to the third period (by comparing A2 and A4 with A1). Moreover, it demonstrates how the power market could have been affected if electricity generators had been treated in the same way as other industries in the EU ETS (by considering A3 and A4).

5 Numerical results

5.1 Base case 2010

Before we examine the effects of the different policies, let us take a brief look at the base case scenario in 2010. Remember that the model is calibrated to the data year 2000. Compared to that year, the model accounts for growth in demand due to economic growth from 2000 to 2010. Moreover, existing capacities in electricity production and transmission in 2000 are depreciated at a fixed annual rate. On the other hand, profitable investments in electricity production and transmission of electricity and natural gas are assumed to be in place in 2010. Still, we should not expect the base case scenario for 2010 to be identical to the actual market situation in 2010, as there are a number of factors (including new policies) influencing the market that is not accounted for in the base case of the model.

In the base case scenario for 2010, CO₂-emissions in Western Europe are 25 percent above the joint annual Kyoto target for the years 2008-12. The electricity sector accounts for about one third of emissions in Western Europe, cf. Table 2 below. The average whole price of electricity is 40 Euro per MWh (see Table 3), with national prices varying in the range 32-43 Euro per MWh (see the Appendix for country details). The average whole prices of steam coal, natural gas and oil are respectively 65, 146 and 249 Euro per toe.

Partly due to relatively low coal prices, coal power production grows substantially compared to 2000, see Table 4 (remember that the base case assumes no new climate policies after 2000). The market share of coal power is thus 37 percent in the base case. Nuclear power, gas power and hydro power have market shares of respectively 24, 21 and 14 percent, whereas other renewables have merely 5

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15 The supply functions for gas to Europe have also been adjusted from 2000 to 2010.
16 Coal and electricity prices in the base case scenario are quite close to actual price levels in 2010, whereas gas and oil prices were significantly higher in reality. Of course, in 2010 energy prices were influenced by the EU ETS, so it may seem more relevant to compare actual 2010-prices with the policy scenarios below.
percent. Oil power is not profitable. Around one quarter of power production in 2010 comes from new plants, i.e., plants built after 2000.

5.2 Case B: A common price of emissions

Assume now that Case B is implemented in Western Europe. This means that CO₂-prices are equal across countries and sectors. Let us first consider the benchmark allocation mechanism A1 (“Unconditional grandfathering”), which mimics the conventional assumption that quotas are allocated in a lump sum way. Then we see from Table 3 that the price of CO₂ must be 33 Euro per tonne in order to comply with the Kyoto target. This is the cost-effective way of complying with the target, and we should expect the welfare costs to be minimized in this scenario (we return to this below). From Table 2 we notice that emissions in the electricity sector are reduced relatively more than in the other sectors, which is not surprising given the larger degree of substitutability in this sector.

Table 2. CO₂-emissions in the electricity sector and other sectors in the base case and under different allocation mechanisms. Case B. Million tonnes per year.

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>1,430</td>
<td>858</td>
<td>874</td>
<td>1,031</td>
<td>898</td>
</tr>
<tr>
<td>Other sectors</td>
<td>2,872</td>
<td>2,589</td>
<td>2,573</td>
<td>2,416</td>
<td>2,549</td>
</tr>
<tr>
<td>Total</td>
<td>4,301</td>
<td>3,447</td>
<td>3,447</td>
<td>3,447</td>
<td>3,447</td>
</tr>
</tbody>
</table>

Table 3. Average wholesale prices of energy goods and uniform CO₂-prices across countries/sectors under different allocation mechanisms. Case B.

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity (€₂₀₁₀/MWh)</td>
<td>40</td>
<td>56</td>
<td>56</td>
<td>47</td>
<td>52</td>
</tr>
<tr>
<td>Coal (€₂₀₁₀/toe)</td>
<td>65</td>
<td>57</td>
<td>57</td>
<td>57</td>
<td>56</td>
</tr>
<tr>
<td>Gas (€₂₀₁₀/toe)</td>
<td>146</td>
<td>167</td>
<td>160</td>
<td>243</td>
<td>200</td>
</tr>
<tr>
<td>CO₂-price (€₂₀₁₀/tonne)</td>
<td>NA</td>
<td>33</td>
<td>36</td>
<td>53</td>
<td>35</td>
</tr>
</tbody>
</table>

The prices of electricity increase as coal and gas power plants get higher marginal operating costs due to the cost of emitting CO₂. Even if coal and gas power producers receive quotas, they treat the quota allocation as a lump sum transfer as long as the allocation is unconditional. The average wholesale price of electricity in Western Europe increases from 40 to 56 Euro per MWh (see Table 3). The price increase is lowest in the Nordic countries, where the shares of coal and gas power plants are much smaller than in the rest of Europe.
The wholesale prices of coal decrease, whereas the natural gas prices increase. The explanation is that the CO₂-price leads to a substantial shift from coal power production to gas power and renewable power production, cf. Table 4. Gas power production increases despite higher operating costs due to the CO₂-price. The reason is that the supply of CO₂-free electricity is limited, and so the price of electricity must increase sufficiently to make gas power production more profitable than without the climate policy. The market share of coal power drops from 37 percent to 16 percent, whereas gas power and non-hydro renewable power rise to 31 and 10 percent, respectively. Investments in gas power and renewable power production are increased significantly compared to the base case situation, while investments in new coal power capacity are no longer profitable.

Table 4. Annual production of electricity by technologies under different allocation mechanisms.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Case</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing plants</td>
<td>2,667 (78%)</td>
<td>2,283 (74%)</td>
<td>2,336 (76%)</td>
<td>2,301 (68%)</td>
<td>1,965 (62%)</td>
</tr>
<tr>
<td>Coal</td>
<td>824 (24%)</td>
<td>506 (16%)</td>
<td>531 (17%)</td>
<td>555 (16%)</td>
<td>491 (15%)</td>
</tr>
<tr>
<td>Gas</td>
<td>465 (14%)</td>
<td>397 (13%)</td>
<td>424 (14%)</td>
<td>357 (11%)</td>
<td>95 (3%)</td>
</tr>
<tr>
<td>Oil</td>
<td>2 (0%)</td>
<td>-</td>
<td>1 (0%)</td>
<td>12 (0%)</td>
<td>-</td>
</tr>
<tr>
<td>Nuclear</td>
<td>831 (24%)</td>
<td>831 (27%)</td>
<td>831 (27%)</td>
<td>831 (25%)</td>
<td>831 (26%)</td>
</tr>
<tr>
<td>Hydro</td>
<td>441 (13%)</td>
<td>441 (14%)</td>
<td>441 (14%)</td>
<td>441 (13%)</td>
<td>441 (14%)</td>
</tr>
<tr>
<td>Other renew.</td>
<td>103 (3%)</td>
<td>108 (4%)</td>
<td>108 (4%)</td>
<td>105 (3%)</td>
<td>107 (3%)</td>
</tr>
<tr>
<td>New plants</td>
<td>755 (22%)</td>
<td>783 (26%)</td>
<td>732 (24%)</td>
<td>1,063 (32%)</td>
<td>1,203 (38%)</td>
</tr>
<tr>
<td>Coal</td>
<td>438 (13%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Gas</td>
<td>237 (7%)</td>
<td>566 (18%)</td>
<td>514 (17%)</td>
<td>940 (28%)</td>
<td>1,030 (33%)</td>
</tr>
<tr>
<td>Oil</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Nuclear</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hydro</td>
<td>22 (1%)</td>
<td>33 (1%)</td>
<td>33 (1%)</td>
<td>27 (1%)</td>
<td>31 (1%)</td>
</tr>
<tr>
<td>Other renew.</td>
<td>59 (2%)</td>
<td>184 (6%)</td>
<td>185 (6%)</td>
<td>95 (3%)</td>
<td>142 (4%)</td>
</tr>
<tr>
<td>Total</td>
<td>3,422 (100%)</td>
<td>3,066 (100%)</td>
<td>3,068 (100%)</td>
<td>3,364 (100%)</td>
<td>3,168 (100%)</td>
</tr>
</tbody>
</table>

The welfare costs of this cost-effective policy are calculated to 29 Billion Euro per year, see Table 5. Note that we disregard environmental benefits from lower CO₂ emissions in these calculations. Consumer surplus and non-energy producer surplus are substantially reduced (in aggregate), even if the increased government surplus were distributed back to these groups. On the other hand, in this policy alternative power producers are better off than without climate policy. The reason is of course the large increase in electricity prices, but also the allocation of free quotas to gas, coal and oil power producers who benefit most from this policy: Gas producers more than triple their profits, while coal producers’ profits increase by more than 150 percent. With full auctioning of quotas, both gas and coal...
producers would stand to lose, but power producers as a group would still gain as producers using CO₂-free energy would increase their profits by 60 percent. Consumers and non-energy producers would still lose out even if they get the revenues from auctioning of quotas.

Table 5. Economic welfare changes compared to Base Case. Case B. Billion Euro per year

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer and non-energy producer surplus</td>
<td>-151</td>
<td>-158</td>
<td>-190</td>
<td>-152</td>
</tr>
<tr>
<td>Power producer surplus</td>
<td>43</td>
<td>44</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>Other energy producer surplus and trade surplus</td>
<td>3</td>
<td>0</td>
<td>22</td>
<td>11</td>
</tr>
<tr>
<td>Government surplus</td>
<td>77</td>
<td>83</td>
<td>115</td>
<td>80</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>-29</strong></td>
<td><strong>-30</strong></td>
<td><strong>-49</strong></td>
<td><strong>-40</strong></td>
</tr>
</tbody>
</table>

Let us now consider the effects of allocation mechanism A2, i.e., “Conditional grandfathering”. As explained in the previous section, the difference compared to A1 (“Unconditional grandfathering”) is that allocated quotas to a power plant are withdrawn if the plant does not maintain all of its capacity. The first order effect of this “threat” is increased incentives to maintain capacity, especially for existing coal power plants, which to a large degree shut down under allocation mechanism A1. Figure 3 shows that total maintained capacity is indeed 9 percent higher under A2 than under A1.

Figure 3. Maintained capacity of electricity generation by technologies under different allocation mechanisms. Case B. GW
With more capacity available, production of electricity from existing plants also increases compared to the cost-effective outcome in A1.\textsuperscript{17} This is seen in Table 6. We notice that especially supply from old coal power plants increases in A2 compared to A1. This crowds out some production from new gas power plants, so that total production of electricity is approximately the same under A1 and A2. The share of coal is slightly higher in A2, whereas the share of gas is slightly lower.

Consistent with almost unchanged production, the price of electricity is also almost the same in A2 as in A1.\textsuperscript{18} The average price of gas decreases by four percent. On the other hand, the price of CO\textsubscript{2} increases by about nine percent as the allocation mechanism stimulates capacity maintenance and thus production (as a first order effect), and so the price must increase in order to comply with the overall target. Moreover, the electricity sector now has a bigger share of total emissions as the higher CO\textsubscript{2}-price reduces emissions in the rest-sector.

The costs of complying with the overall emissions target increase by five percent (relative to A1) when the quota allocation to power plants is conditioned on maintained capacity. Consumers and non-energy producers are worse off due to a higher CO\textsubscript{2}-tax, but this has its counterpart in higher government tax revenues. Gas producers lose profits as reduced gas power production leads to lower gas prices, whereas power plants using lignite are the main winners among power producers. The higher total welfare costs are due to the fact that more expensive power plants (accounting for the shadow costs of CO\textsubscript{2}-emissions) are employed to produce almost the same amount of electricity. The cost differential is moderated by the fact that there are beneficial terms-of-trade effects for Western Europe as whole in A2, related to lower import prices of natural gas (compared to A1). The opposite is the case in A3 and A4, cf. the gas prices in Table 3.

The first two allocation mechanisms allocate quotas based on historic emissions. The other two mechanisms allocate quotas based on plants’ current activity levels, i.e., in proportion to either output (A3) or (maintained) capacity (A4). Consequently, the latter two mechanisms give enhanced incentives to respectively produce (A3) or maintain/invest in power capacity (A4) for the fossil-based plants that receive quotas.

\textsuperscript{17} Not all the maintained capacity is used, however. For instance, oil power capacities are maintained in all countries under A2, but only used in one country.

\textsuperscript{18} Both total production and the average price increase marginally, which may seem inconsistent. The explanation is that the average price is a weighted average over four time periods and 16 regions. Thus, the price falls in some periods/regions and increases in other periods/regions.
The first order effect of allocation mechanism A3 (compared to A1) is to increase electricity production in coal and gas power plants. This leads to lower electricity prices, whereas the price of CO₂ must increase in order to comply with the overall emissions target. These price effects reduce the profitability of coal and gas power plants, and the higher CO₂-price is clearly most harmful for (old) coal power producers. As a consequence, output-based allocation (A3) is most favourable for gas power plants, which explains why gas power increases its market share from 31 percent (under A1) to 39 percent (under A3), cf. Table 4. This is consistent with the analytical results derived in Section 2, showing that the least emission-intensive producers will benefit from output-based allocation.

The increased gas power production comes from new gas power plants, whereas old gas power plants actually produce less than in A1. The reason is that new plants are more effective than existing plants, and hence emit less CO₂ per unit kWh produced. The market share of coal power is about the same, as the effect of higher CO₂-price more or less matches the effect of increased production incentives. Renewable electricity production drops, however, due to lower electricity prices (see below). Increased investments in gas power capacity crowds out investments in especially wind power capacity. Total power production increases by 10 percent compared to A1. The CO₂-price ends up 60 percent higher in A3 than in A1, which is also consistent with the analytical results in Section 2.

As indicated already, the price of electricity is reduced even more in A3 compared to A1 and A2; in A3 the price of electricity is 15 percent lower than in A1. There are two reasons why the price is lower (and production higher) in A3 than in A2. First, A3 has the highest price of CO₂, which is due to the strong incentives to produce and thus emit under this allocation mechanism. A higher CO₂-price means that less emissions take place in the rest-sector, and thus more emissions are allowed in the electricity sector (see Table 2). Second, the emission-intensity is reduced in A3 compared to A2, as allocation mechanism A2 mostly favour old coal power plants, while allocation mechanism A3 mostly favour new gas power plants. Combined, these two factors imply that production is higher and electricity prices lower in A3 than in A2.

The economic welfare costs of allocation mechanism A3 are significantly higher than under A1 and A2, i.e., 70 percent higher than under A1. This might seem counter-intuitive inasmuch as more and cleaner electricity is produced under A3. The explanation is that the implicit subsidy provided by the allocation mechanism A3 stimulates more supply from more expensive (though somewhat cleaner) power plants. Someone has to pay for these higher costs of supply, and we notice that the group of power producers is significantly worse off with this allocation mechanism despite the implicit production subsidy, see Table 5. Even the group of gas producers loses profits despite the large expansion of gas power, as new gas power plants to some degree crowd out old plants. Consumers are
also worse off under A3 than under A1, but not if extra government surplus is distributed to the consumers. Then they are more or less indifferent, due to a combination of lower electricity prices and substantially higher gas prices (vis-à-vis A1).

The last allocation mechanism A4 makes it more profitable to maintain existing capacity and invest in new capacity, but not necessarily to produce electricity. Still, when more capacity is available, production will typically also increase as the marginal cost of producing one more unit is lower when investment costs are sunk. As shown in Figure 3, total power production capacity is indeed highest under this allocation mechanism. In particular, investments in new gas power plants are increased relative to all other scenarios, including A3. In addition, much more of the existing coal and gas capacity are maintained.

From Table 4 we notice that total electricity production increases slightly compared to A1 and A2 (grandfathered allocation), i.e., by three percent, but is lower than under output-based allocation (A3), which stimulates production directly. Production from new gas power plants is, however, higher under A4 than under A3, whereas production from existing gas and coal power plants is reduced compared to all other scenarios. The reduction is particularly strong for existing gas power plants. The reason is that increased supply from new gas power plants harms existing gas power plants (which are less efficient than the new ones) via three channels: Higher gas prices, higher CO2-prices, and lower electricity prices.

The price of CO2 under A4 lies between the price level under A1 and A2. The price of electricity, however, is six percent lower than under A1 and A2, which is consistent with the higher level of production. The welfare costs are substantially lower with allocation mechanism A4 than with A3. The main reason is the much higher CO2 price under A3, which substantially shifts emission reductions from the electricity sector to the other sectors (compare with the cost-effective solution A1 in Table 2). Nevertheless, the welfare costs are 40 percent higher than under A1, mostly because it leads to excessive levels of maintained power capacity.

Looking at the increase in the average electricity price (relative to base case) and the CO2-price under the four different allocation mechanisms, we notice that the relationship between these two varies a lot. Under the cost-effective allocation mechanism A1, the ratio is 0.47 tonne CO2 per MWh.19 The corresponding ratios under A2, A3 and A4 are 0.43, 0.13 and 0.34 respectively. This illustrates quite clearly that the so-called pass-through of CO2-prices into the electricity price does not only depend on

19 \((56 \text{ €/MWh} - 40 \text{ €/MWh}) / (33 \text{ €/ton CO2}) = 0.47 \text{ ton CO2 per MWh}\)
whether coal or gas power (old or new) is on the margin, but also on the way quotas are allocated to
the power producers, working through the multimarket equilibria, notably the markets for fuel inputs.
With output-based allocation, the pass-through rate (0.13) is in fact only one-third of the emission
intensity of new gas power plants (0.38 tonne CO₂ per MWh), and one sixth of the emission intensity
of new coal power plants (0.78 tonne CO₂ per MWh).

5.3 Case A: Fixed sectoral emissions

Assume now instead that Case A is implemented, which means that a certain share of the overall
emissions target is allocated to the electricity sector, irrespective of allocation mechanism. This share
is set equal to the sector’s share of emissions in the base year, i.e., 27 percent. We notice from Table 2
that the share is lower than in the base case scenario of 2010 (33 percent), but it is slightly higher than
in the cost-effective emission reduction scenario described above (Case B-A1), where the electricity
sector accounted for 25 percent of emissions.

Most of the qualitative insight reached above for Case B, such as the differences between the four
allocation mechanisms, carries over to Case A. Thus, we will not go into all the details here. We will
rather highlight the main differences between Case A and B, and refer the interested reader to the
tables in Appendix B.

With lump sum allocation of quotas (A1: “Unconditional grandfathering”), the price of CO₂ in the
electricity sector is about 29 Euro per tonne (see Table 6), i.e., 13 percent lower than with equal CO₂-
price across all sectors (Case B-A1). On the other hand, for the other sectors the weighted average
price of CO₂ across Western Europe is 50 Euro per tonne. The explanation is of course that it is less
costly to reduce emissions in the power sector, where more substitution possibilities are present. This
is well known to policy makers in Europe, who seem to put more ambitious targets on this sector than
on other sectors. Note, however, that the CO₂ price for the other sectors varies quite substantially
across countries due to different emission targets and different costs of reducing emissions (e.g.,
because of differences in fuel mix and industry structure).

Under allocation mechanism A3 the CO₂ price is higher in the electricity sector than in the rest of the
economy, and we notice from Table 6 that the quota price is more than twice as high under A3 than
under A1. In the Case A scenarios, emissions in the electricity sector cannot increase, and thus the
price of emissions has to increase even more under Case A-A3 than under Case B-A3.
Although total emissions in the rest-sector in each country are fixed across allocation mechanisms (Case A), we notice from Table 6 that the average CO₂ tax for these sectors is affected quite much by the allocation mechanism used in the electricity sector. The reason is that changes in market shares and prices in the electricity market affect the rest of the economy as well. In particular, as electricity prices fall (due to more power production) and gas prices increase (due to more gas power) under A3 and A4 vis-à-vis A1, end users have increased incentives to switch from fossil fuels (especially gas) to electricity, and thus the necessary CO₂-price to attain the given emission target decreases.

Table 6. CO₂-prices under different allocation mechanisms. Case A. €2010 per tonne CO₂

<table>
<thead>
<tr>
<th>CO₂-price</th>
<th>Electricity sector</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other sectors⁵</td>
<td>50</td>
<td>50</td>
<td>43</td>
<td>47</td>
</tr>
</tbody>
</table>

⁵ Weighted average over all countries.

Not surprisingly, welfare costs become higher when CO₂-prices are not harmonized across sectors and countries. Costs are increased by almost 37 percent under Case A-A1 compared to Case B-A1, see Table 7. In particular, consumers are worse off. On the other hand, we notice that the welfare costs under Case A-A1 are significantly lower than under Case B-A3, i.e., choosing output-based allocation, but keeping CO₂ prices equal across sectors and regions, is more costly than letting the electricity sector pay less than 60 percent of the CO₂ price in other sectors. Furthermore, the welfare costs of Case A-A3 are only slightly higher than that of Case B-A3, even though the former scenario has one additional distortion compared to the latter scenario. One reason is that the Case B-A3 scenario not only creates distortions within the electricity sector, but also leads to too much emission reductions in the other sectors (cf. the high CO₂-price under A3 in Table 3). Thus, in a way the fixed allocation between the electricity sector and the rest of the economy under Case A lessens to some degree the distortions created by the output-based allocation.

Table 7. Economic welfare changes compared to Base Case. Case A. Billion Euro per year

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer surplus</td>
<td>-195</td>
<td>-197</td>
<td>-180</td>
<td>-186</td>
</tr>
<tr>
<td>Power producer surplus</td>
<td>40</td>
<td>42</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>Other prod.surplus</td>
<td>3</td>
<td>1</td>
<td>26</td>
<td>11</td>
</tr>
<tr>
<td>Government surplus</td>
<td>112</td>
<td>114</td>
<td>97</td>
<td>106</td>
</tr>
<tr>
<td>Total</td>
<td>-39</td>
<td>-40</td>
<td>-51</td>
<td>-49</td>
</tr>
</tbody>
</table>
6 Conclusions

The main insight from this paper is that the choice of allocation mechanism, i.e., how to allocate emission quotas to firms regulated by an emission trading system, can have substantial price and quantity effects. Moreover, the welfare costs of reaching a fixed emission target may increase significantly if certain allocation mechanisms are chosen. Although our numerical analysis applies to the electricity sector of Western Europe, the qualitative insight has relevance also for other regions and other industries.

Whenever the allocation of quotas is conditioned on firms’ activity in some way or another (e.g., level of production or maintained capacity), firms’ incentives vis-à-vis this activity are changed. Typically, this activity is stimulated, usually resulting in higher production and emissions. As long as the overall emission target is fixed, the price of quotas is then driven up. The product market (in our case the electricity market) is also affected, but the impacts on this market depend highly on the choice of allocation mechanism. Some allocation mechanisms favour old over new installations, whereas others favour less emission-intensive over more emission-intensive technologies. For instance, we find that output-based allocation of quotas (A3) may significantly strengthen the substitution from coal power to gas power. Furthermore, capacity-based allocation (A4) may boost investments in new gas power plants.

In general, allocation mechanisms that lead to large deviations from the cost-effective outcome with regards to prices and quantities also imply much larger welfare costs of reaching a fixed emission target. This is especially true for output-based allocation. Choosing this allocation mechanism can in fact be more costly for society than having considerably different CO₂-prices across sectors.

The numerical model we have used is a static equilibrium model, and hence allocation of quotas has been linked to current activity in our analysis. Allocation of quotas in the EU ETS, however, is to a large degree linked to past activity. However, as the base year for allocation has been regularly updated (e.g., allocation to industry installations in the years 2013-20 will be based on installations’ production in 2007-8), firms may think that higher activity today may provide more free quotas in the future. If so, the allocation mechanisms analyzed within our static model framework is merely a simplification of the actual allocation mechanisms, and firms’ incentives are changed in similar ways in the real world as in the model. However, both uncertainty about future allocation mechanisms and discounting of future benefits could imply that the model overstates the differences between the allocation mechanisms if allocation is linked to past instead of current activity. On the other hand, uncertainties about future allocation rules may generate additional distortions.
As explained above, the electricity sector will as a general rule no longer receive quotas in the EU ETS from 2013, but most of the new member states (EU-12) will nevertheless continue to allocate quotas to their power installations. Our welfare results may provide some indications about the benefits of departing from free allocation to this sector, or alternatively, the costs of continuing this practice. Note, however, that the allocation mechanism used for this sector in the EU ETS is not the most costly one in our analysis (A3), but rather a mixture of two others (A2 and A4). On the other hand, with different allocation rules across countries, additional inefficiencies may arise.

The remaining sectors of the EU ETS will still receive substantial amounts of quotas, using a mixture of allocation mechanism A3 and A4. Our analysis may suggest that this could be quite costly for the EU compared to pure auctioning. However, the objective of free allocation to these sectors is to prevent carbon leakage (cf. Böhringer et al., 2010). Thus, it is difficult to judge in general whether the extra costs showing up in our analysis are higher or lower than the eventual benefits of reduced leakage. Still, our results suggest that free allocation of quotas to sectors with little or no carbon leakage is suboptimal.

References


http://energycommerce.house.gov/Press_111/20090701/hr2454_house.pdf


Appendix A

Assume that there are two types of firms in the quota sector. To simplify notation, we consider only two firms, firm 1 and firm 2, but all results carry over to the case with several identical firms of type 1 and several identical firms of type 2. Let \( c_i(y_i) = z(y_i) + q_i x_i(y_i) \) where \( q_i \) is the (per assumption) common price of fuels (USD/toe) for firms in sector 1. To simplify, we assume that firms have the same non-fuel cost function, \( z(y_i) \), which is assumed to be increasing and convex. Because \( x_i(y_i) \) is increasing and convex, see discussion above, also the cost function \( c_i(y_i) \) is increasing and convex.

Below we assume that firm 1 uses at least as much fossil fuels as firm 2 in order to attain a specific production level, that is, \( x_{11} = \alpha x_{11} \) and \( x_{21} = x_{21} \), where \( \alpha \geq 1 \). Thus, \( c_i'(y_{11}) = z'(y_{11}) + q_i \alpha x_i'(y_{11}) \) and \( c_i'(y_{21}) = z'(y_{21}) + q_i x_i'(y_{21}) \). In the discussion below we will examine two sources of heterogeneity; either that the input requirement differs between the two firms in sector 1 (\( \alpha > 1 \)), that is, firm 1 is more fossil-fuel intensive than firm 2, or that the emission coefficient differs between firms in this sector, that is, \( \omega_{11} > \omega_{21} \).

As noted in Section 2, relations (2), (4) (for \( s = 1 \)), (5) (for \( s = 1 \)), (6), (7) and (8) (for \( s = 1 \)) determine \( y_{11}, p_1, \kappa_1, D_1, T_1 \) and \( \gamma^0 \) given the exogenous variables \( \gamma, g \) and \( e_1 \). In fact, relations (2), (4) (for \( s = 1 \)), (5) (for \( s = 1 \)), (7) and (8) (for \( s = 1 \)) determine \( y_{11}, p_1, \kappa_1, D_1 \) and \( T_1 \) given the exogenous variables \( \gamma, g \) and \( e_1 \), whereas \( \gamma^0 \) is then determined from (6). Taking this observation into account, we first differentiate (5) (for \( s = 1 \)) wrt. \( \gamma \), which gives us a relationship between \( \frac{dy_{11}}{d\gamma} \) and \( \frac{dy_{21}}{d\gamma} \):

\[
\frac{dy_{11}}{d\gamma} = -\frac{\omega_{21} x_{21}'(y_{21})}{\omega_{11} \alpha x_{11}'(y_{11})} \frac{dy_{21}}{d\gamma}. \tag{12}
\]

Next, we substitute (4) (for \( s = 1 \)) into (2) and then differentiate (2) wrt. \( \gamma \), and finally replace \( \frac{dy_{11}}{d\gamma} \) with (12). The derived system of equations then has the form...
\[
\begin{align*}
A \frac{dy_{21}}{d\gamma} + B \frac{d\kappa_i}{d\gamma} &= -\kappa_i \\
C \frac{dy_{21}}{d\gamma} + D \frac{d\kappa_i}{d\gamma} &= -\kappa_i
\end{align*}
\]  \quad (13)

where \( B = -\left(\omega_{11}x'(y_{11}) - \gamma \right) < 0 \), \( D = -\left(\omega_{21}x'(y_{21}) - \gamma \right) < 0 \) and

\[
C - A = \left(\omega_{11}x'(y_{11}) + (q_1 + \kappa\omega_{21})x_{q}(y_{21})\right) - \left(\omega_{21}x'(y_{21}) + (q_1 + \kappa\omega_{11})x_{q}(y_{11})\right) < 0.
\]

Solving (13) we find

\[
\frac{dy_{21}}{d\gamma} = \frac{\kappa_i(B - D)}{AD - BC} - \frac{\kappa_i(-\omega_{11}x'(y_{11}) + \omega_{21}x'(y_{21}))}{AD - BC}.
\]  \quad (14)

In order to find the sign of \( \frac{dy_{21}}{d\gamma} \) we need to know the sign of \( AD - BC \). We know from Proposition 1 that \( \frac{d\kappa_i}{d\gamma} > 0 \). From (13) we find that \( \frac{d\kappa_i}{d\gamma} = \kappa_i(C - A)/(AD - BC) \). Hence, the sign of \( AD - BC \) must equal the sign of \( C - A \), which is negative, see right after (13). Hence, \( AD - BC \) is negative.\(^{20}\)

From (2) we have

\[
\left(q_1 + \kappa\omega_{11}\right)\alpha x_{q}'(y_{11}) + z'(y_{11}) = \left(q_1 + \kappa\omega_{21}\right)x_{q}'(y_{21}) + z'(y_{21}).
\]  \quad (15)

Assume first that emission coefficients do not differ between firm 1 and firm 2 (\( \omega_{11} = \omega_{21} = \omega \)), and that \( \alpha > 1 \). Consider first the special case of \( z'(y_{11}) = 0 \), i.e., linear non-fuel costs.\(^{21}\) From (15) we then find

\[
\alpha x_{q}'(y_{11}) = x_{q}'(y_{21}).
\]  \quad (16)

---

\(^{20}\) As an alternative strategy to find the sign of \( AD - BC \), one can simply use the expressions for \( A, B, C \) and \( D \). Tedious calculations reveal that \( AD - BC < 0 \).

\(^{21}\) Although here referred to as a special case, we believe that linear non-fuel costs may be just as realistic as strictly convex non-fuel costs in the electricity sector; the numerical equilibrium model LIBEMOD, which is used in Section 5, has linear non-fuel costs in the electricity sector.
Using (14) we find that \( \frac{dy_{21}}{d\gamma} = 0 \), and thus \( \frac{dy_{11}}{d\gamma} = 0 \) from (12). A marginal increase in \( \gamma \) shifts, cet. par., the marginal cost curve of firms 1 and 2 downwards by the same magnitude \( (\kappa_i) \), and hence the relative position of the two firms do not change. The shift in \( \gamma \) leads to a higher quota price \( \kappa_i \) (see above), which will shift the marginal cost curves upwards, but again the two shifts are identical, reflecting relation (16). Thus, in the new equilibrium production at the firm level has not changed. This proves the first part of Proposition 2.

Next, assume that non-fuel costs are strictly convex (when emission coefficients do not differ between firm 1 and firm 2 and \( \alpha > 1 \)). From (15) it then follows that \( y_{21} > y_{11} \) and thus \( z'(y_{21}) > z'(y_{11}) \). Consequently, we must now have

\[
\alpha x'_i(y_{11}) > x'_i(y_{21}). \tag{17}
\]

Using (14) under restriction (17), we find \( \frac{dy_{21}}{d\gamma} > 0 \), and thus \( \frac{dy_{11}}{d\gamma} < 0 \) from (12). Hence, with strictly convex non-fuel costs, the most efficient firm increases its production whereas production in the least efficient firm decreases, which is consistent with our conjecture. In this case, the marginal cost curve of the inefficient firm shifts more upwards than the marginal cost curve of the efficient firm when the price of quotas increases, reflecting \( \alpha x'_i(y_{11}) > x'_i(y_{21}) \). Finally, using (12) we find the effect on total production in the quota sector:

\[
\frac{dy_1}{d\gamma} = \frac{dy_{11}}{d\gamma} + \frac{dy_{21}}{d\gamma} = \frac{\omega_1 \alpha x'_i(y_{11}) - \omega_2 x'_i(y_{21})}{\omega_1 \alpha x'_i(y_{11})} \frac{dy_{21}}{d\gamma}.
\tag{18}
\]

In the present case the nominator in (18) equals \( \omega(\alpha x'_i(y_{11}) - x'_i(y_{21})) \), which is positive under (17).

Hence, \( \frac{dy_1}{d\gamma} > 0 \). This concludes the proof of the last part of Proposition 2.

We now consider the case of different emission coefficients and \( \alpha = 1 \). Assume that \( \omega_{11} > \omega_{21} \), that is, also in this case firm 1 is the “inefficient” one. From (15) it then follows that \( y_{21} > y_{11} \), and thus \( x'_i(y_{21}) > x'_i(y_{11}) \), and \( z'(y_{21}) \geq z'(y_{11}) \) (equality if the non-fuel cost function is strictly convex). We then rewrite (15) and impose \( \alpha = 1 \):

\[
q_i(x'_i(y_{11}) - x'_i(y_{21}))) + (z'(y_{11}) - z'(y_{21})) = \kappa_i(\omega_{12} x'_i(y_{21}) - \omega_{11} x'_i(y_{11})). \tag{19}
\]
From the discussion above we know that the left hand side of (19) is negative, and hence the right hand side of (19) also has to be negative:

$$\omega_2 x'_1(y_{21}) < \omega_1 x'_1(y_{11}).$$

(20)

Thus in the present case the nominator in (14) is negative and the nominator in (18) is positive. Hence, $$\frac{dy_{21}}{d\gamma} > 0$$ and $$\frac{dy_1}{d\gamma} > 0.$$ Finally, using (12) we have $$\frac{dy_{11}}{d\gamma} < 0.$$ This concludes the proof of Proposition 3.

We now turn to the case of a common price of emissions in the two sectors. This case is defined by relations (2) - (10), see discussion above. In order to find the equilibrium effects of a shift in \( \gamma \), we first use (10) to replace \( \kappa_1 \) and \( \kappa_2 \) with \( \kappa \). Next, combining (5) - (9) and differentiating wrt. \( \gamma \) we find

$$\omega_2 x'_1(y_{21}) \frac{dy_{21}}{d\gamma} - \omega_2 x'_2 \frac{dy_2}{d\gamma} = \omega_1 x'_1(y_{11}).$$

(21)

Next, substituting (4) (for \( s = 2 \)) into (3) and differentiating, we find a relationship between $$\frac{dy_2}{d\gamma}$$ and $$\frac{d\kappa}{d\gamma}$$, which is then substituted into (21):

$$\omega_2 x'_1(y_{21}) \frac{dy_{21}}{d\gamma} - \omega_2 x'_2 \frac{dy_2}{d\gamma} = \omega_1 x'_1(y_{11}) \frac{(\omega_2 x'_2)^2}{d\gamma}. \frac{d\kappa}{d\gamma}.$$

(22)

We then substitute (4) (for \( s = 1 \)) into (2) and differentiate, and then use (22) to substitute for $$\frac{dy_{11}}{d\gamma}.$$ The resulting system of equations has the same form as (13), but with different coefficients; \( \bar{A}, \bar{B}, \bar{C} \) and \( \bar{D} \). We find that

$$\frac{dy_{21}}{d\gamma} = \frac{\kappa(\bar{B} - \bar{D})}{\bar{A} \bar{D} - \bar{B} \bar{C}} = \kappa \left( \frac{-\omega_1 x'_1(y_{11}) + \omega_2 x'_1(y_{21}) + (\omega_2 x'_2)^2(q_1 + \kappa \omega x'_2(y_{11}))}{\omega_1 x'_1(y_{11})(P'_2 - c_2^\alpha - \kappa \omega x'_2(y_{11}))} \right) \frac{\omega_1 x'_1(y_{11})(P'_2 - c_2^\alpha - \kappa \omega x'_2(y_{11}))}{\bar{A} \bar{D} - \bar{B} \bar{C}}.$$

(23)
Further, using (22) we find that the effect on total production in the quota sector \((y_1 = y_{11} + y_{21})\) is

\[
\frac{dy_1}{dy} = \frac{(\omega_{11}x'(y_{11}) - \omega_{21}x'(y_{21}))}{\omega_{21}x'(y_{11})} \frac{dy_{21}}{dy} - \frac{(\omega_{21}x'(y_{21})^2}{\omega_{21}x'(y_{11})} \frac{d\kappa}{dy}
\]

(24)

In order to sign the denominator in (23), we use the same type of argument as in the case of fixed sectoral emissions: Solving the system of equations we find \(\frac{d\kappa_1}{dy} = \kappa_1(\bar{C} - \bar{A})/(\bar{AD} - \bar{BC})\). From Proposition 4 we know that this derivative is positive. Moreover, it can be shown that \(\bar{C} - \bar{A} < 0\) and hence \(\bar{AD} - \bar{BC} < 0\).

We now consider the different cases of heterogeneous firms. Assume first that emission coefficients do not differ between firm 1 and firm 2 \((\omega_{11} = \omega_{21} = \omega)\), and that \(\alpha > 1\). Consider first the special case of \(z^*(y_{11}) = 0\), that is, \(\alpha x'_1(y_{11}) = x'_2(y_{21})\), see (16). Then the first term in the nominator of (23) is zero. The second term in the nominator of (23) is always negative. Because we know that the denominator in (23) is negative, see discussion above, we have shown that \(\frac{dy_{21}}{dy} > 0\). Further, in our case the first term in the nominator of (24) is zero, whereas the second term is always positive and the denominator is positive. Hence, \(\frac{dy_1}{dy} > 0\). Finally, from (22) we see that the first term in the nominator is negative (this is the same term as in the case of fixed sectoral emissions), whereas the second term is positive (this term reflects lower emissions in the tax sector, which opens up for more emissions, and thus more production, in firm 1). Hence, the impact on production of firm 1 is ambiguous.

Next, consider the case of non-fuel costs being strictly convex when emission coefficients do not differ between firm 1 and firm 2, and \(\alpha > 1\). We then have \(\alpha x'_1(y_{11}) > x'_2(y_{21})\), see (17). Then the first term in the nominator of (23) is negative, and the first term in the nominator of (24) is positive. From the discussion above it then follows that \(\frac{dy_{21}}{dy} > 0\) and \(\frac{dy_1}{dy} > 0\). Again, the impact on production of firm 1 is ambiguous, see (22).

Finally, consider the case of different emission coefficients \((\omega_{11} > \omega_{21})\) and \(\alpha = 1\). We then have \(\omega_{21}x'_1(y_{21}) < \omega_{11}x'_1(y_{11})\), see (20). Like in the previous case the first term in the nominator of (23) is
negative, and the first term in the nominator of (24) is positive. Hence, \( \frac{dy_{21}}{d\gamma} > 0 \), \( \frac{dy_1}{d\gamma} > 0 \) and the impact on production of firm 1 is ambiguous. This concludes the proof of Proposition 5.
Appendix B

Case B

Table A1. Electricity (node) prices under different allocation mechanisms. Case B. $/MWh

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
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<td>47</td>
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<td>59</td>
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<td>55</td>
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<td>57</td>
<td>57</td>
<td>47</td>
<td>53</td>
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<td>57</td>
<td>48</td>
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<tr>
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<td>56</td>
<td>46</td>
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Case A

Table A2. Average wholesale prices of energy goods under different allocation mechanisms. Case A

<table>
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<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity ($/MWh)</td>
<td>40</td>
<td>54</td>
<td>55</td>
<td>49</td>
<td>52</td>
</tr>
<tr>
<td>Coal ($/toe)</td>
<td>65</td>
<td>58</td>
<td>58</td>
<td>57</td>
<td>57</td>
</tr>
<tr>
<td>Gas ($/toe)</td>
<td>146</td>
<td>167</td>
<td>162</td>
<td>255</td>
<td>200</td>
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</tbody>
</table>
Table A3. Annual production of electricity by technologies under different allocation mechanisms. Case A. TWh (Market shares in parenthesis)

<table>
<thead>
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<th>Base Case</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
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<tr>
<td>Coal</td>
<td>824 (24%)</td>
<td>571 (18%)</td>
<td>579 (19%)</td>
<td>470 (14%)</td>
<td>504 (16%)</td>
</tr>
<tr>
<td>Gas</td>
<td>465 (14%)</td>
<td>405 (13%)</td>
<td>433 (14%)</td>
<td>347 (11%)</td>
<td>94 (3%)</td>
</tr>
<tr>
<td>Oil</td>
<td>2 (0%)</td>
<td>0 (0%)</td>
<td>3 (0%)</td>
<td>12 (0%)</td>
<td>-</td>
</tr>
<tr>
<td>Nuclear</td>
<td>831 (24%)</td>
<td>831 (27%)</td>
<td>831 (27%)</td>
<td>831 (25%)</td>
<td>831 (26%)</td>
</tr>
<tr>
<td>Hydro</td>
<td>441 (13%)</td>
<td>441 (14%)</td>
<td>441 (14%)</td>
<td>441 (13%)</td>
<td>441 (14%)</td>
</tr>
<tr>
<td>Other renew.</td>
<td>103 (3%)</td>
<td>107 (3%)</td>
<td>107 (3%)</td>
<td>106 (3%)</td>
<td>107 (3%)</td>
</tr>
<tr>
<td><strong>New plants</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coal</td>
<td>438 (13%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>22 (1%)</td>
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<tr>
<td>Gas</td>
<td>237 (7%)</td>
<td>579 (18%)</td>
<td>521 (17%)</td>
<td>932 (28%)</td>
<td>1,032 (32%)</td>
</tr>
<tr>
<td>Oil</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Nuclear</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hydro</td>
<td>22 (1%)</td>
<td>32 (1%)</td>
<td>32 (1%)</td>
<td>29 (1%)</td>
<td>30 (1%)</td>
</tr>
<tr>
<td>Other renew.</td>
<td>59 (2%)</td>
<td>167 (5%)</td>
<td>173 (6%)</td>
<td>113 (3%)</td>
<td>138 (4%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>3,422 (100%)</td>
<td>3,134 (100%)</td>
<td>3,120 (100%)</td>
<td>3,280 (100%)</td>
<td>3,199 (100%)</td>
</tr>
</tbody>
</table>

Figure A1. Maintained capacity of electricity by technologies under different allocation mechanisms. Case A. GW