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Pumped-Storage Hydroelectricity

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Pumped-Storage Hydroelectricity*

by

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Abstract: Pumped-storage hydroelectricity has been proposed as one of the solutions to the non-storability of intermittent energy. The basic economics of pumped storage is explored using thermal generation, pure intermittent energy and general hydropower "topped up" with pumped-storage hydroelectricity. The implications of using pumped storage for trade in electricity between a hydro country and a foreign market and between a hydro country and an intermittent country both with exogenous and endogenous prices are analysed.

JEL classification: L12, Q25, Q32, Q42

Keywords: Pumped-storage hydroelectricity; Intermittent energy, Thermal generation; Hydropower; Trade

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1. Introduction

World-wide efforts to reduce emissions of climate gasses have led many countries to pursue a policy of increasing the share of renewable energy in order to move away from a carbon-based generation of electricity. Renewables - wind power, solar and small-scale hydro power - are intermittent and uncontrollable and therefore needs other generating technologies to undertake the necessary adjustment of supply in order to keep the continuous balance between demand and supply. A crucial question is the ability to store intermittent energy. An idea that has been floated in European media is that the reservoirs of hydropower plants in Norway and Sweden can serve as battery storage for Europe. The idea is that surplus wind power can be absorbed by the hydro system simply by reducing the current use of stored water, and then exporting back when wind power is scarce.

The recent decision to close down nuclear plants in Germany has led to an increased emphasis on ambitious plans for investing in renewables like wind and solar in Germany. These plans have been accompanied with an expressed German interest in pumped storage in hydro-rich countries like Norway (SRU, 2010). Pumped storage increases the amount of stored water over a yearly period, and hence increases the ability of hydro reservoirs to serve as a battery for countries producing a high share of intermittent energy.

The standard pumped storage consists of a source of water (river, lake) at the location of the generator and a purpose- built reservoir at a higher altitude without any natural inflow (Wikipedia, Pumped-storage hydroelectricity). Water can be pumped up to the reservoir and then released on to the turbines to generate electricity. The world-wide capacity installed so far is rather limited and made for supply adjustment of the daily cycle. However, equipping existing hydropower plants with turbines that can be converted to pumps means that huge reservoirs already in place can be used, and seasonal demand cycles can then also be met (Warland et al., 2011).

The topic of pumped-storage hydroelectricity is traditionally an engineering one, with numerous papers in technical journals on the topic. Economists have not shown that much interest. However, because less energy is created than the energy it takes to pump up water, there is an economic problem at the heart of pumped storage. The fundamental requirement for pumped storage being an economic proposition is that there must be a price difference between periods of sufficient magnitude so the loss is overcome by the difference in price, and in addition there is the cost of the investment in pumped storage to be covered. There are also both technical and economic problems involved with other generating technologies being forced to be swing producers due to the intermittency of renewable energy.

The plan of the paper is revisit the economics of pumped storage in Section 2 in the case of thermal generation in addition to pumped storage. In Section 3 only intermittent generation is studied together with pumped storage, and in Section 4 a hydropower country with the possibility of using pumped storage is opened up to trade with exogenous trading prices. Section 5 introduces endogenous trading prices trade between two countries where one country has only hydropower with reservoirs and the other country has only intermittent power. The impact of trade for the economics of pumped storage in the hydro country is explored. Section 6 concludes.

2. Thermal generation and pumped storage

In a recent paper Crampes and Moreaux (2010) study the use of pumped storage together with thermal electricity generation within a region (country) without external links. This model will be the point of departure. The problem of investment in capacity is not studied (Horsley and Wrobel, 2002). A two-period model is used as in the original paper. To extend the analysis to multiple periods is not so straightforward. The reasons for this will be commented upon below.

As pointed out in Crampes and Moreaux an early economics paper on pumped storage is Jackson (1973). The motivation for studying pumped storage there was that the generation of electricity was done by nuclear power, and this technology should be run as base load both for technical and economic reasons. Therefore, daily cycles in demand can better be met by pumped storage.

A detailed specification of various thermal technologies will not be pursued. The costs of running thermal capacity, c_t, is expressed by an aggregate cost function

$$c_t = c(e_t^{Th})\ (c_t' > 0, c_t'' > 0,\ e_t^{Th} \le \overline{e}^{Th}), t = 1,2 \tag{1}$$

The output of thermal electricity during a period *t* is e_t^{Th} measured in an energy unit (kWh), and $\overline{e}^{Th}$ is the upper capacity limit. It is assumed that this cost function reflects a unique merit order of using the individual generators and that there are no connections between costs between periods, i.e. start-up and close-down costs are ignored. The technology is stationary over the periods, and the costs of primary fuels stay constant (Førsund, 2007, Chapter 5).

The production function for the pumped storage is a traditional hydro power production function (Førsund, 2007) depending on the head (level difference between the reservoir and the generator) and the amount of water released onto the turbine. The amount of water instantaneously released is either restricted by the capacity of the pipes or by installed turbine capacity. The total amount of water in the reservoir has an upper limit. Considering only two periods (e.g. two seasons within a year) it is common to assume that the reservoir can be completely filled in the first period and emptied in the next period. With a finer time resolution this may no longer be a tenable assumption. Furthermore, it is assumed that in the period when water is pumped up into the reservoir no electricity is produced.

Demand functions on inverse form for electricity is used to evaluate consumption of electricity.[1] This is a standard way of formulating an optimisation problem for a social planner having consumer plus producer surplus as his objective function. The optimal prices are used to evaluate the surpluses, i.e. the areas under the demand functions subtracted variable generating costs. The model is partial and do not have any interactions with the rest of the economy and both the

[1] In Crampes and Moreaux (2010) utility functions are used. Measuring marginal utility in money, demand functions represent marginal utility functions, so to compare results prices can be substituted for marginal utilities in Crampes and Moreaux (2010).

thermal production side and the demand side is aggregated to single systems (Førsund, 2007). The social planner's optimisation problem is:

$$
\begin{aligned}
& Max \sum_{t=1}^{2} [\int_{z=0}^{x_t} p_t(z)dz - c(e_t^{Th})] \\
& \text{subject to} \\
& x_1 = e_1^{Th} - e_1^P \\
& x_2 = e_2^{Th} + e_2^P \\
& e_1^P = \mu e_2^P \,, \mu > 1 \\
& e_2^P \le \overline{e}^P \\
& e_t^{Th} \le \overline{e}^{Th} \,, t = 1,2 \\
& x_t, e_t^{Th}, e_t^P \ge 0 \,, t = 1,2 \\
& \overline{e}^P, \overline{e}^{Th} > 0
\end{aligned}
\qquad (2)
$$

The first two conditions state the energy balances. The electricity used for pumping is e_1^P and the hydroelectricity generated is e_2^P. The conditions have to hold with equality since there must be balance between supply and demand in continuous time for a well-behaved electricity system. The third condition links the amount of electricity used for pumping in the first period to the amount of hydro electricity generated in the second period. Because we only have one period when water can be released after pumping up, all water, if there is any pumping-up in period 1, will be produced in a single period; period 2. (In a multi-period setting the economic point is, of course, the control of the period when to release the pumped water going for the highest difference.) Pumped storage consumes more electricity than it generates, as indicated by the restriction on the parameter μ>1. The pumping operation faces three constraints; the capacity of the pump itself, the capacity of the pipe for the water transport up to the reservoir, and the capacity of the reservoir of the system. We will assume that only one constraint can cover these possibilities and constrain the water to be stored by the upper limit $\overline{e}^P$. The amount of water pumped up is e_1^P / μ and the water to be stored is e_2^P and these are equal. The next two conditions state the capacity limits of the thermal production system, and then we have the non-negativity conditions.

The availability of the pumped storage facility makes the optimisation problem (2) in general a dynamic problem. Prices and quantities for both periods must be solved simultaneously.

In order to simplify the derivation of the first-order conditions we substitute from the energy balances inserting the expressions for the consumption variables, and eliminate the electricity for pumping as a separate variable when forming the Lagrangian for the optimisation problem (2):[2]

$$\begin{aligned}
L = & \int_{z=0}^{e_1^{Th}-\mu e_2^P} p_t(z)dz + \int_{z=0}^{e_2^{Th}+e_2^P} p_t(z)dz - \sum_{t=1}^{2} c(e_t^{Th}) \\
& -\sum_{t=1}^{2} \gamma_t^{Th}(e_t^{Th} - \bar{e}^{Th}) \\
& -\gamma^P(e_2^P - \bar{e}^P)
\end{aligned} \tag{3}$$

The necessary first-order conditions are:

$$\begin{aligned}
& \frac{\partial L}{\partial e_1^{Th}} = p_1(e_1^{Th} - \mu e_2^P) - c'(e_1^{Th}) - \gamma_1^{Th} \le 0 \ (= 0 \text{ for } e_1^{Th} > 0) \\
& \frac{\partial L}{\partial e_2^{Th}} = p_2(e_2^{Th} + e_2^P) - c'(e_2^{Th}) - \gamma_2^{Th} \le 0 \ (= 0 \text{ for } e_2^{Th} > 0) \\
& \frac{\partial L}{\partial e_2^{H}} = -\mu p_1 + p_2 - \gamma^P \le 0 \ (= 0 \text{ for } e_2^P > 0) \\
& \gamma_t^{Th} \ge 0 (= 0 \text{ for } e_t^{Th} < \bar{e}^{Th}), t = 1,2 \\
& \gamma^P \ge 0 (= 0 \text{ for } e_2^P < \bar{e}^P)
\end{aligned} \tag{4}$$

We will make the reasonable assumption that electricity is produced in both periods. Assuming no satiation of demand implies then that prices are positive. The expression μp_1 is the price in period 1 marked up with the factor showing the amount of electricity needed in period 1 to produce a unit of electricity in period 2. We will call this expression the loss-corrected price.

The third condition in (4) for use of the pumped storage facility tells us:

i) When the price in period 2 is strictly less than the loss-corrected price in period 1 then pumped storage is not used:

   $p_2 < \mu p_1 \Rightarrow e_2^P = 0$

[2] In Crampes and Moreaux (2010) the hydro production in period 2 is chosen as the variable to substitute. The qualitative conclusion will, of course, be the same.

(We may have equality in the third expression and no pumping, but we disregard this possibility.) According to the complementary slackness condition for the Lagrangian parameter we have that $\gamma^P = 0$ since we do not generate hydroelectricity in period 2.

ii) When the price in period 2 is equal to the loss-corrected price in period 1 we have that the pumped storage facility typically will be used to some extent; we have an interior solution

$$p_2 = \mu p_1 \Rightarrow 0 < e_2^P < \overline{e}^P .$$

We have that $\gamma^P = 0$ because the capacity is not constrained.

iii) When the price in period 2 is greater than the loss-corrected price in period 1 we have that the pumped storage facility is used to its full capacity:

$$p_2 = \mu p_1 + \gamma^P \Rightarrow p_2 > \mu p_1$$

We have typically $\gamma^P > 0$ when the capacity constraint is binding.

The general condition for not using the pumped storage facility by pumping up water in period 1 and producing hydroelectricity in period 2 (pumping up water in period 1 and not using it in period 2 obviously cannot be part of an optimal solution) is that $\mu p_1 - p_2 \geq 0$ (from the complementary slackness condition we have that $\gamma^P = 0$). The typical condition is that the loss-corrected price in period 1 is greater than the price in period 2. The optimal price difference, $p_2 - p_1$, is not big enough to warrant using the pumped storage facility.

If hydroelectricity is produced in the second period then it may seem that it is formally not necessary that thermal is utilised in period 2. But for thermal not to be utilised in the second period we must have $p_2 \leq c'(0)$. However, this creates a contradiction because we must have $c'(0) < p_1$ in the first period because thermal capacity is used, and for production of hydro to be optimal in period 2 we must have $p_1 < p_2$. After all, pumped storage is used just to increase electricity consumption in period 2. We must therefore produce electricity both from thermal generators and pumped storage. The price in period 2 is lower with the use of pumped storage than without. Therefore less thermal capacity is used than without pumped-storage hydroelectricity.

Notice that without using the pumped storage facility there is no connection between the periods. The optimal solution for each period is found solving static optimisation problems for each period separately.

Assuming an interior solution and a use of the pumped storage facility we have from the first-order conditions

$$\begin{aligned} &p_1(e_1^{Th} - \mu e_2^P) = c'(e_1^{Th}) \\ &p_2(e_2^{Th} + e_2^P) = c'(e_2^{Th}) \\ &\mu p_1 = p_2 \end{aligned} \tag{5}$$

The optimal prices are equal to the marginal cost of thermal in each period, and the loss-corrected price in the pumping period is equal to the price in the second period when the water is processed. An estimate of the factor μ is indicated to be in the interval 1.15-1.30 (Wikipedia, Pumped-storage hydroelectricity). The relationship between the optimal prices implies an analogous relationship between the marginal costs in the two periods; $\mu c'(e_1^{Th}) = c'(e_2^{Th})$. This implies that thermal generation must be higher in period 2 than in period 1, confirming the fact that thermal generation will be used in period 2 (cf. the discussion above). Equality of the prices between periods or equality of marginal costs of thermal generation will never be optimal in an interior solution.

If the constraint on thermal capacity is binding a shadow price is added. This may occur in the peak period, and most unlikely in the off-peak period, remembering that the price in the peak period when hydro is used must be higher than in the previous pumping-up period. (However, it is technically possible to have binding thermal capacity constraints in both periods. The shadow prices on the thermal capacity will then differ between the periods because the marginal cost at full capacity utilization is the same in both periods, but the price in period 1 must be smaller than the price in period 2.)

In the case that the reservoir is constrained a shadow price will be switched on in the last first-order condition in (4). This implies a greater gap between the prices of the two periods than in the unconstrained case, as also shown in point iii) above:

$$p_2 - p_1 = p_1(\mu - 1) + \gamma^P \tag{6}$$

If more storage capacity would have been available more water would be pumped up into the reservoir in the first period and more hydro would be produced in the second period thus reducing the price gap due to an increased price in the first period and a reduced price in the second.

The optimal interior solution (5) is illustrated in Figure 1 using two quadrants[3]. Period 1-consumption is measured to the left of the central price- and marginal cost axis erected vertically from the origin *O*. Period 2 consumption is measured to the right. The marginal cost functions are identical and are drawn as straight lines upwards to the left and right from the common anchoring point at $c'(0)$ on the central axis. The short vertical lines at the end of the marginal cost curves indicate the limited capacity. The demand curves are also straight lines for ease of

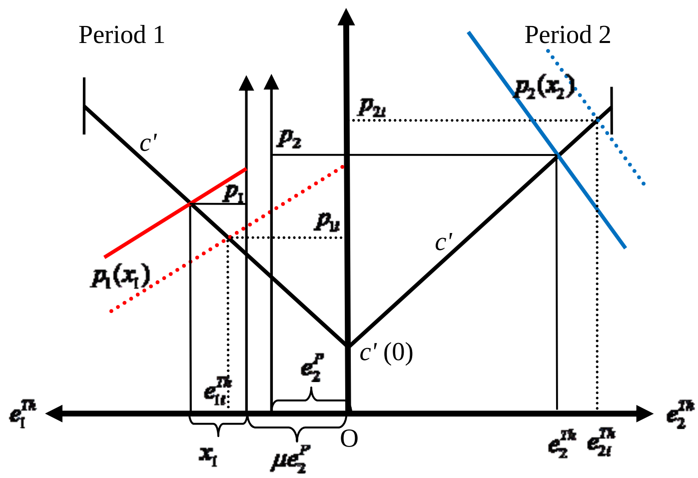

Figure 1. Optimal use of thermal power and pumped storage

exposition. The capacity limit of the hydro reservoir is not shown in order not to overload the illustration, but can be introduced as a vertical line to the left of μe_2^P.

[3] The illustration is different from the illustrations found in Crampes and Moreaux (2010).

The situation with only using thermal generation is shown with dotted demand curves. Period 1 demand is made more elastic with a considerably lower choke price, resulting in a lower price and quantity than the situation for period 2, (coordinates (p_{1i}, e_{1i}^{Th}) and (p_{2i}, e_{2i}^{Th}) respectively, marked by subscript *i* for independent system as in Crampes and Moreaux, 2010) indicated by the thin horizontal and vertical dotted lines. Without loss of generality period 2 is the high-demand period (peak) and period 1 the low-demand period (off-peak). Period 1 may be called summer and period 2 winter, or night and day.

In the case of the pumped-storage facility being used the necessary generation of thermal electricity, μe_2^P, for pumping up water in order to produce e_2^P units of hydroelectricity in period 2 is shown in the period 1 quadrant. The difference $\mu e_2^P - e_2^P = (\mu - 1)e_2^P$ is the physical loss of electricity incurred in the transformation of thermal electricity into hydro-generated electricity. The demand curve for general consumption is shifted to the left and is now anchored on the vertical line up from μe_2^P on the left-hand horizontal axis. The intersection of the shifted demand curve, drawn with a solid line, and the marginal cost curve, results in the consumption price p_1 and the quantity e_1^{Th}. The demand curve for period 2 is also shifted to the left, due to the energy axis for period 2 shifting to the left to the vertical up from the point e_2^P, and drawn solid. We get the price-quantity combination ($p_2, e_2^P + e_2^{Th}$). As a check that the first-order condition for optimality in Eq. (5) is obeyed the relative difference between electricity needed in period 1 to produce the illustrated amount of hydro in period 2 should be the same as the relative difference between the optimal prices, and equal to the relative difference between marginal costs of thermal generation in the two periods. This is roughly the case in the illustration.

In the illustration the consumption of electricity decreases in the first period when electricity is used to pump up water and total thermal production is increased, but consumption increases in the second period when the water is processed, although the thermal production is contracted due to the lower price. All these changes are general features if it is optimal to use the pumped storage, and follow from diverting thermal electricity in the first period to pump water, and the addition of hydro production in the second period.

The consumer plus producer surplus is clearly going down in period 1 from the isolated thermal case to using pumped storage, illustrated by the larger surplus triangle in the former case than in the latter case. In period 2 the consumer price is reduced and the quantity increased so the consumer surplus is clearly greater in the case of using pumped storage than in the isolated thermal case, but it is a little more difficult to see what happens with the change in costs. In the illustration the reduction in thermal costs in period 2 seems to be about the same as the generation costs incurred in period 1 due to pumping up water. In any case we know that the social benefit has increased if the figure is an illustration of the optimal solution. The loss of social benefit in period 1 must typically be more than outweighed by the increased social benefit in period 2. If this is not the case pumped storage will not be used in an optimal solution.

Summing up the results, we have that pumped-storage hydroelectricity reduces the difference in price between the two periods by increasing the price in the low-price period and decreasing the price in the high-price period. But it is never optimal to have these prices equal. The price difference in an interior solution implies that the loss of electricity due to the pumping activity is just offset by the price difference at the margin. The value of the electricity used for pumping in the low-price period is more than compensated by the gain of hydroelectricity in the high-price period.

Generalising to many periods

The optimisation problem (1) may be generalised to many periods, *T*, in the following way:

$$
\begin{aligned}
&Max \sum_{t=1}^{T} [\int_{z=0}^{x_t} p_t(z)dz - c(e_t^{Th})] \\
&\text{subject to} \\
&x_t = e_t^{Th} - e_t^P , t = 1,...,T-1 \\
&x_{t+j} = e_{t+j}^{Th} + e_{t+j}^P , \ j = 1,...,T-t, \ t = 1,...,T-1 \\
&e_t^P = \mu e_{t+j}^P , \mu > 1 \\
&e_{t+j}^P \le \bar{e}^P \\
&e_t^{Th} \le \bar{e}^{Th} , t = 1,2 \\
&x_t, e_t^{Th}, e_1^P, e_2^P \ge 0, t = 1,2, \ \bar{e}^P, \bar{e}^{Th} > 0
\end{aligned}
\tag{1'}
$$

We assume that pumping up takes place in period t, and that the water is released onto the turbines in period $t+j$, where the index j may take on values making the production period be any period from $t+1$ to the last period T. The simplest case is that pumping-up only takes place once and also the production of hydroelectricity. However, this is hardly a realistic generalization to many periods. There may be several periods with pumping-up, and several periods with production of hydroelectricity. If we keep the simplifying assumption that the pumping reservoir takes one period to fill up we have in principle to inspect all pairwise combinations of pumping-up periods and periods producing hydroelectricity that fulfills the third condition in (4) that the loss-adjusted price in the pumping-up period is greater or equal to the price in the later period of producing hydroelectricity. A complication is that after a pumping-up period the water has to be produced before a new pumping period can take place and the same goes for periods producing hydroelectricity. This restriction of at least one period in between each of these activities must be entered. A special algorithm is needed to find the optimal number of active periods and the exact timing.

If it should take more than a period to fill the reservoir the situation becomes more complicated. Depending on the capacity of the reservoir relative to the definition of a period length (the number of periods it takes to fill the reservoir will of course differ between a period length of one hour and one day or week) the building-up of water in the reservoir may take a number of pumping-up periods. It may also take more than one period to empty the reservoir. It is not so obvious how one should go about to find a solution to the optimisation problem with these extensions.

3. Intermittent power and pumped storage

The case of only having intermittent energy may be realistic for isolated regions like islands where links to the central grid of the country in question are too expensive. The use of pumped storage can be analysed using the model in the previous section substituting thermal generation for intermittent generation:

$$
\begin{aligned}
& Max \sum_{t=1}^{2} [\int_{z=0}^{x_t} p_t(z) dz] \\
& \text{subject to} \\
& x_1 = e_1^I - e_1^P \\
& x_2 = e_2^I + e_2^P \\
& e_1^P = \mu e_2^P \,, \mu > 1 \\
& e_2^P \le \bar{e}^P \\
& e_t^I = a_t \bar{e}^I \,, a_t \in [0,1],\ t = 1,2 \\
& x_t, e_t^I, e_t^P \ge 0 \,, t = 1,2 \\
& \bar{e}^P, \bar{e}^I > 0
\end{aligned} \tag{7}
$$

The modelling of intermittent energy follows Førsund and Hjalmarsson (2011). It is assumed that there are no variable costs producing the intermittent energy e_t^I. The intermittent generation of electricity is represented by a time-dependent coefficient converting wind, sunshine or run-of-the-river water into energy based on the installed power capacity[4]. The coefficient, reflecting the average availability of the primary energy source for intermittent energy (e.g. wind conditions) may take the value between zero and a maximal value based on full utilisation of the power capacity for the period that is normalised to 1 (i.e. corresponding to the wind strength sufficient to fully utilise the installed capacity).[5] Furthermore, we assume that available production is always used, implying an equality sign in the "production function" in the last relation in (7). Substituting from the energy balances for consumption we are left with only one energy decision variable; the amount of electricity to produce by pumped storage in the second period. The Lagrangian function for the optimisation problem (7) is

$$
\begin{aligned}
L = & \int_{z=0}^{e_1^I - \mu e_2^P} p_t(z) dz + \int_{z=0}^{e_2^I + e_2^P} p_t(z) dz \\
& - \gamma^P (e_2^P - \bar{e}^P)
\end{aligned} \tag{8}
$$

---

[4] If a disaggregation is wanted each intermittent technology can be described using the production relation in (1) (Førsund, 2012).

[5] In a disaggregated framework the distribution of the coefficient is site –specific.

The necessary first-order condition is:

$$\frac{\partial L}{\partial e_2^P} = -\mu p_1(e_1^I - \mu e_2^P) + p_2(e_2^I + e_2^P) - \gamma^P \le 0 \; (=0 \text{ for } e_2^H > 0)$$
$$\gamma^P \ge 0 (=0 \text{ for } e_2^P < \bar{e}^P) \tag{9}$$

The amounts of exogenous intermittent energy appear in the demand functions and have an influence on the solution via these. The interior solution with $\gamma^P = 0$ is:

$$\mu p_1(e_1^I - \mu e_2^P) = p_2(e_2^I + e_2^P) \tag{10}$$

Knowing the demand functions and the intermittent generations this equation can be solved for pumped hydro in the second period and then the prices follow. If the price difference between period 1 and 2 cannot be realised, i.e. the cost-adjusted price in period 1 is greater than the price in period 2, pumped storage will not be used. Another corner solution is that the reservoir for pumped water will be filled. Then, as in the case for thermal power in the previous section, a positive shadow price on the reservoir constraint adds to the required price difference.

The optimal interior solution (10) is illustrated in Figure 2 using two quadrants, following Fig.1.

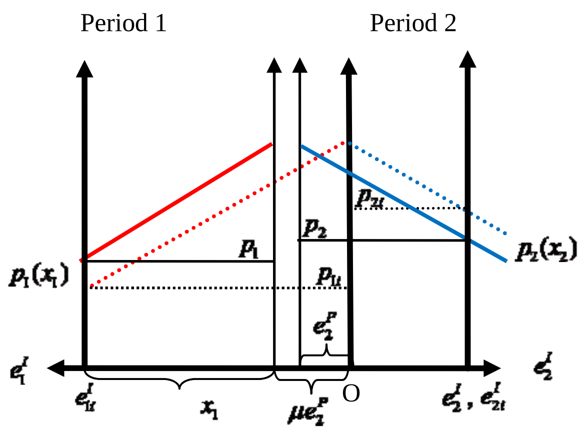

Figure 2. Optimal use of intermittent energy and pumped storage

The demand functions are assumed to be equal to highlight the impact of different intermittent energy between the two periods. The demand curves for the situation without using the pumped-storage facility are the dotted lines yielding the period prices p_{1i} and p_{2i}, where sub-index i indicates no use of pumped storage. When pumped storage is used, the given intermittent energy for period 1 is split between consumption in period 1 and the use for pumping, μe_2^P. The axis for the residual consumption demand is moved correspondingly to the left up from μe_2^P. The residual demand curve is shown by the solid line, and the price in period 1 must increase to p_1. In period 2 the hydroelectricity e_2^P is added to the intermittent energy e_2^I resulting in the energy axis for period 2 moving to the left from O to the line up from e_2^P. The corresponding shift of the demand curve to the left is shown by the solid line. The price is lowered to p_2. Given that the pumped storage is used without constraining the reservoir the relative difference between the period prices should correspond to the cost mark-up factor μ.

4. Hydropower and pumped storage with trade to exogenous prices

To discuss the issue of hydropower in some countries serving as a battery for other countries with a high share of intermittent energy we need a model encompassing trade in electricity. As a start we will assume that the electricity production in a hydro-rich country, or more precisely the volume of the trade, is not big enough to influence the price in an intermittent-rich country, implying that the hydro country can take the trading prices as given. The loss of electricity due to the transport between the countries is disregarded for simplicity. The capacity of the interconnectors plays an important role setting the limit for the amounts that can be traded. We stick to the format of two periods, and open for the possibility that the hydro country has the option to enhance the reservoir's capacity to produce electricity in the second period by pumped storage.

The hydro sector will be modelled as an aggregate sector with only a constraint on the total storage capacity of water (Førsund, 2007). R_t is the amount of water at the end of period t (t =1, 2), $\bar{R}$ is the storage capacity (the maximal amount of water that can be used; the lower level is for simplicity normalised to zero), w_t is the inflow of water during period t and e_t^H is the production in period t.

The social planner's optimisation problem is:

$$
\begin{aligned}
& Max \sum_{t=1}^{2} [\int_{z=0}^{x_t} p_t(z)dz + p_t^{XI} e_t^{XI})] \\
& \text{subject to} \\
& x_1 = e_1^H - e_1^P - e_1^{XI} \\
& x_2 = e_2^H + e_2^P - e_2^{XI} \\
& e_1^P = \mu e_2^P \\
& \frac{e_1^P}{\mu} = e_2^P \le \bar{e}^P \\
& R_1 \le R_o + w_1 + e_1^P / \mu - e_1^H \\
& R_2 \le R_1 + w_2 - e_2^P - e_2^H \\
& R_t \le \bar{R} \\
& -\bar{e}^{XI} \le e_t^{XI} \le \bar{e}^{XI} \\
& x_t, e_t^H, e_1^P, e_2^P, w_t, R_t, \ge 0, t = 1, 2 \\
& \bar{R}, \bar{e}^P, \bar{e}^{XI} > 0, \mu > 1
\end{aligned}
\qquad (11)
$$

Income from exports is added to the consumer plus producer surplus in the objective function and expenses of import subtracted. A balance in trade of electricity is not imposed. The first two relations in the constraint set are the energy balances. New variables are the (net) traded amounts $e_t^{XI} (t = 1, 2)$. When electricity is imported it is negative, while export is positive. The corresponding exogenous prices are $p_t^{XI} (t = 1, 2)$. As in the previous models capacity for pumping is limited to $\bar{e}^P$. Because the pumping facility (e.g. equipped with a reversible turbine) is using the existing reservoir for the hydro system it is logical to connect the constraint to the ability to pump water up. However, due to the third relation in (11) we can still express the constraint by constraining the production of hydroelectricity from the pumping facility in the second period.

There are three relations describing the hydropower system, the two first describing the water accumulation and the third giving the capacity constraint of the reservoir. When this constraint holds with equality we may have overflow. All variables are actually measured in energy units (kWh) although the expression water is used.

The reservoir capacity is not influenced by the pump storage capacity, but pumping means that the inflow to the reservoir in the first period increases. In a model with an aggregated hydro sector it is logical to assume that in the period the pumping takes place hydropower cannot be used, but only in the next period. The electricity in the pumping period must therefore exclusively be based on import.

The Lagrangian function for the problem (11) is:

$$\begin{aligned}
L = & \int_{z=0}^{e_1^H - \mu e_2^P - e_1^{XI}} p_1(z)dz + \int_{z=0}^{e_2^H + e_2^P - e_2^{XI}} p_2(z)dz + p_1^{XI} e_1^{XI} + p_2^{XI} e_2^{XI} \\
& -\gamma^P (e_2^P - \bar{e}^P) \\
& -\lambda_1 (R_1 - R_o - w_1 - e_2^P + e_1^H) \\
& -\lambda_2 (R_2 - R_1 - w_2 + e_2^P + e_2^H) \\
& -\sum_{t=1}^{2} \gamma_t (R_t - \bar{R}) \\
& -\sum_{t=1}^{2} \alpha_t (e_t^{XI} - \bar{e}^{XI}) \\
& -\sum_{t=1}^{2} \beta_t (-e_t^{XI} - \bar{e}^{XI})
\end{aligned} \tag{12}$$

The two last expressions identify whether the export or the import is constrained in a period.

The first-order conditions are:

$$
\begin{aligned}
&\frac{\partial L}{\partial e_t^H} = p_t - \lambda_t \leq 0 \ (= 0 \,\text{for}\, e_t^H > 0) \\
&\frac{\partial L}{\partial e_2^P} = -\mu p_1 + p_2 - \gamma^P + \lambda_1 - \lambda_2 \leq 0 \ (= 0 \,\text{for}\, e_2^P > 0) \\
&\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \,\text{for}\, R_t > 0) \\
&\frac{\partial L}{\partial e_t^{XI}} = -p_t + p_t^{XI} - \alpha_t + \beta_t = 0 \\
&\gamma^P \geq 0 (= 0 \,\text{for}\, e_2^P < \bar{e}^P) \\
&\lambda_1 \geq 0 (= 0 \,\text{for}\, R_1 < R_o + w_1 + e_2^P - e_1^H) \\
&\lambda_2 \geq 0 (= 0 \,\text{for}\, R_2 < R_1 + w_1 - e_2^P - e_1^H) \\
&\gamma_t \geq 0 (= 0 \,\text{for}\, R_t < \bar{R}) \\
&\alpha_t \geq 0 (= 0 \,\text{for}\, e_t^{XI} < \bar{e}^{XI}) \\
&\beta_t \geq 0 (= 0 \,\text{for} -e_t^{XI} < \bar{e}^{XI})
\end{aligned} \tag{13}
$$

Pumping can only occur in period 1 and then pumped water is used in period 2. If pumping then period 1 must be an import period and no water is used. We must assume that sufficient reservoir capacity is available. From the first condition in (13) we have:

$$p_1 < \lambda_1 , p_2 = \lambda_2 = \lambda_1 \tag{14}$$

If the pumping facility is used we see from the second condition in (13):

$$-\mu p_1 + p_2 - \gamma^P + \lambda_1 - \lambda_2 = 0 \Rightarrow \mu p_1 = p_2 - \gamma^P \tag{15}$$

The shadow price on pumping capacity is the change in the objective function of a marginal increase in the pumping capacity. The maximal gain without any net loss in electricity caused by pumping is the price in period 2 so the difference on the right-hand side is positive.

We assume no overflow or threat of overflow in period 1. From the fourth condition we have the connections between domestic prices and trade prices

$$p_1 = p_1^{XI} + \beta_1 , \ \ p_2 = p_2^{XI} - \alpha_2 \tag{16}$$

Putting together the last two equations yields

$$\mu(p_1^{XI} + \beta_1) = (p_2^{XI} - \alpha_2) - \gamma^P \tag{17}$$

We then have the following inequality between the domestic prices expressed by the trade prices and shadow prices on transmission capacity:

$$(p_1^{XI} + \beta_1) = \frac{1}{\mu}(p_2^{XI} - \alpha_2 - \gamma^P) \Rightarrow (p_1^{XI} + \beta_1) < (p_2^{XI} - \alpha_2) - \gamma^P \tag{18}$$

Period 1 price must be sufficiently smaller that period 2 price, and with a binding constraint on pumping capacity the price difference must not only compensate for the net loss of electricity when pumping ($\mu > 1$), but the difference will be larger and reflect the shadow price on the pumping capacity.

Assuming an interior solution for pumped-storage hydroelectricity Eq. (18) shows the role of the constraints on the interconnector for the question of whether using the pumping facility is optimal or not. Because the shadow price on the interconnector capacity is added to the import price to form the domestic price in period 1 and subtracted from the export price in period 2 constraining the interconnector works against the condition for using the pumping facility.

An interior solution for trade but corner solution for pumping capacity yields

$$\mu p_1^{XI} = p_2^{XI} - \gamma^P \tag{19}$$

The relations between the exogenous trading prices must then satisfy

$$p_1^{XI} = \frac{1}{\mu}(p_2^{XI} - \gamma^P) \Rightarrow p_1^{XI} < p_2^{XI} - \gamma^P \tag{20}$$

If the loss-corrected domestic price in period 1 is greater than the price in period 2 subtracted the shadow price on the pumped-storage hydroelectricity this capacity cannot be fully utilised. Furthermore, if the loss-corrected domestic price is greater than the price in period 2 the pumping facility will not be used.

5. Trade between countries Hydro and Intermittent with endogenous prices

Two countries are introduced, one country using only hydropower to generate electricity and the other only using intermittent energy. We may think about Norway as the hydropower country and Germany as the intermittent country. For the latter country this is in accordance with long-term plans for carbon-free generation of electricity (SRU, 2010). The variables for the countries are marked with super- and subscripts H and I respectively. When two trading countries cooperate the imports and exports cancel out in the objective function. It is assumed that no income-distributional issues are linked to the trade in electricity in the model.

The optimisation problem for the cooperative problem is

$$
\begin{aligned}
& Max \sum_{t=1}^{2} [\int_{z=0}^{x_{Ht}} p_{Ht}(z)dz + \int_{z=0}^{x_{It}} p_{It}(z)dz)] \\
& \text{subject to} \\
& x_{H1} = e_1^H - e_1^P + e_{I1}^{XI} - e_{H1}^{XI} \\
& x_{H2} = e_2^H + e_2^P + e_{I1}^{XI} - e_{H2}^{XI} \\
& e_1^P = \mu e_2^P \\
& \frac{e_1^P}{\mu} = e_2^P \le \bar{e}^P \\
& R_1 \le R_o + w_1 + e_1^P / \mu - e_1^H \\
& R_2 \le R_1 + w_2 - e_2^P - e_2^H \\
& R_t \le \bar{R} \\
& x_{I1} = e_1^I + e_{H1}^{XI} - e_{I1}^{XI} \\
& x_{I2} = e_2^I + e_{H2}^{XI} - e_{I2}^{XI} \\
& e_t^I = a_t \bar{e}^I , a_t \in [0,1] \\
& e_{Ht}^{XI} \le \bar{e}^{XI} \\
& e_{It}^{XI} \le \bar{e}^{XI} \qquad\qquad (21) \\
& x_{Ht}, x_{It}, e_t^H, e_t^P, e_t^I, e_{Ht}^{XI}, e_{It}^{XI}, w_t, R_t \ge 0, t = 1,2 \\
& \bar{R}, \bar{e}^P, \bar{e}^I, \bar{e}^{XI} > 0, \mu > 1
\end{aligned}
$$

The modelling follows Førsund (2007); (2011). Because one country's export is the other country's export we only need to consider export variables from the two countries in the model. The two first constraints are the energy balances for the hydro country, and then the two next conditions specify the pumping facility. The hydro generation with water storage and upper limit on water storage is covered by the next three constraints. The energy balances for the intermittent country then follows together with the production function for the intermittent power. Lastly the upper constraints on the export variables due to the interconnector between the two countries are specified.

Simplifying by eliminating the variable for electricity for pumping in the first period, e_1^P, and the consumption in the two countries in both periods, the Lagrangian for the optimisation problem (21) is

$$\begin{aligned}
L = &\int_{z=0}^{e_1^H - \mu e_2^P + e_{I1}^{XI} - e_{H1}^{XI}} p_{H1}(z)dz + \int_{z=0}^{e_2^H + e_2^P - e_2^{XI}} p_{H2}(z)dz + \sum_{t=1}^{2}[\int_{z=0}^{e_t^I + e_{Ht}^{XI} - e_{It}^{XI}} p_{It}(z)dz] \\
&-\gamma^P(e_2^P - \overline{e}^P) \\
&-\lambda_1(R_1 - R_o - w_1 - e_2^P + e_1^H) \\
&-\lambda_2(R_2 - R_1 - w_2 + e_2^P + e_2^H) \\
&-\sum_{t=1}^{2}\gamma_t(R_t - \overline{R}) \\
&-\sum_{t=1}^{2}\alpha_{Ht}(e_{Ht}^{XI} - \overline{e}^{XI}) \\
&-\sum_{t=1}^{2}\alpha_{It}(e_{It}^{XI} - \overline{e}^{XI})
\end{aligned} \tag{22}$$

Intermittent energy is assumed to be given exogenously and not subject to optimisation.

The first-order conditions are

$$
\begin{aligned}
&\frac{\partial L}{\partial e_t^H} = p_{Ht} - \lambda_{Ht} \le 0 \ (= 0 \text{ for } e_t^H > 0) \\
&\frac{\partial L}{\partial e_2^P} = -\mu p_{H1} + p_{H2} - \gamma^P + \lambda_1 - \lambda_2 \le 0 \ (= 0 \text{ for } e_2^P > 0) \\
&\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0) \\
&\frac{\partial L}{\partial e_{Ht}^{XI}} = -p_{Ht} + p_{It} - \alpha_{Ht} = 0 \\
&\frac{\partial L}{\partial e_{It}^{XI}} = p_{Ht} - p_{It} - \alpha_{It} = 0 \\
&\gamma^P \ge 0 (= 0 \text{ for } e_2^P < \bar{e}^P) \\
&\lambda_1 \ge 0 (= 0 \text{ for } R_1 < R_o + w_1 + e_2^P - e_1^H) \\
&\lambda_2 \ge 0 (= 0 \text{ for } R_2 < R_1 + w_1 - e_2^P - e_1^H) \\
&\gamma_t \ge 0 (= 0 \text{ for } R_t < \bar{R}) \\
&\alpha_{Ht} \ge 0 (= 0 \text{ for } e_{Ht}^{XI} < \bar{e}^{XI}) \\
&\alpha_{It} \ge 0 (= 0 \text{ for } e_{It}^{XI} < \bar{e}^{XI})
\end{aligned}
\tag{23}
$$

Pumping-up can only occur in period 1 and if it does then the pumped water will be used in period 2. If pumping-up then period 1 must be an import period and no water is used by assumption if the hydro system is fully converted to pumped storage. We come back to the case of pumped storage being a separate facility. We assume no overflow or threat of it in period 1. From the first condition we have:

$$
p_{H1} \le \lambda_1 , p_{H2} = \lambda_2 = \lambda_1 \tag{24}
$$

The price in Hydro in the pumping-up period 1 is typically lower than the water value for period 1. The water values are equal across periods and equal to the price in Hydro in period 2.

If pumped storage is a separate facility then if hydro is used in period 1 at the same time as we have imports and pumping-up this must imply that the import price must be equal to the price in period 1 in Hydro and the water values for the two periods will be the same, and, hence, also the prices (unless there is a threat of overflow in period 1, but it cannot be optimal to have such a threat caused by pumping). But this cannot be part of an optimal solution. So with endogenous prices it is necessary to have no use of water in Hydro in the pumping-up period.

The relationship between the prices in the two countries Hydro and Intermittent follows from the two last first-order conditions. The impact of a constrained interconnector is of special interest. Using pumped storage implies that Hydro is importing in period 1 and exporting in period 2. This implies that, because export from Hydro in period 1 is zero, the active condition is:

$$p_{H1} - p_{I1} - \alpha_{I1} = 0 \Rightarrow p_{H1} \geq p_{I1} \tag{25}$$

The price in Hydro is typically greater than the price in Intermittent in period 1 when the export from Intermittent to Hydro is constrained.

In period 2 the situation is reversed and we have:

$$-p_{H2} + p_{I2} - \alpha_{H2} = 0 \Rightarrow p_{H2} \leq p_{I2} \tag{26}$$

The price in Hydro is typically lower than the price in Intermittent because the export from Hydro is now constrained. Combining our findings we have that

$$p_{H2} \geq p_{H1} \Rightarrow p_{H2} > p_{I1} \text{ and } p_{I1} < p_{I2}\,, \tag{27}$$

the last two strict inequalities being typical results.

The condition for using pumped storage is

$$-\mu p_{H1} + p_{H2} - \gamma^{P} + \lambda_1 - \lambda_2 = 0 \Rightarrow \mu p_{H1} = p_{H2} - \gamma^{P} \tag{28}$$

The water values cancel out. The loss-corrected price in Hydro in period 1 must be less than the price in Hydro in period 2. A sufficient price difference between the two periods may be created without the interconnector being constrained due to the effect of using no water in period 1. If the pumping-up capacity is constrained an even larger price difference is required due to the positive shadow price on pumping capacity.

If import to Hydro is constrained in the first period this means that the price in Hydro will increase; Hydro wants to import more than what is feasible. There will still be a price difference in Hydro between the two periods if no use of water in period 1 remains optimal, but it will be smaller than in a situation with unconstrained import. If import to Hydro is not constrained in period 1 but export from Hydro is constrained in the second period, then the price in Hydro in period 2 will be lower than in the unconstrained case; Hydro would have exported more, but now more electricity has to be consumed at home. The price difference will become smaller between the two periods. If the interconnector is constrained in both periods, then the price difference shrinks “at both ends” compared with the unconstrained case.

6. Conclusions

Pumped-storage hydroelectricity offers a way of storing energy for redistribution over periods. It has been used on a somewhat limited scale to dampen the price differences between peak load and off-peak load demand periods complementing thermal power with a pumped storage facility. More recently pumped storage has been proposed in European context on a large scale for increasing the production of electricity based on existing reservoirs of water for hydropower plants in Scandinavia.

To capture the basic economics of pumped storage a number of models combining this with other generating technologies are presented based on two periods only. The “classical” case of thermal power extended with pumped storage is first analysed and the condition for use of pumped storage found and illustrated in a novel way. The rule for use of a pumping facility that the loss-corrected price in the pumping-up period must be less than or equal to the price in the period hydroelectricity is produced by the pumping facility is derived. This rule also holds for combing intermittent power with pumped storage and combining regular hydropower with pumped storage.

In the case of trading opportunities in electricity between countries, one with dominating hydro power with reservoirs and the other with a large share of intermittent, a new element of the constraint on the interconnector between countries enters the picture. The main result with endogenous trading prices is that if the interconnector becomes constrained this works against the requirement of a sufficient price difference between the pumping period and the production period. Large-scale expansion of interconnectors between countries with different technologies promotes trade, and also makes the use of pumped storage more favourable. The necessary price difference between the periods for pumping to take place is due to no water being used in the hydro-dominated country when importing form the intermittent-dominated country. However, for this to take place as sufficient reservoir capacity has to be assumed.

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